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BULLETIN NO. 62

THE ELECTRON THEORY OF MAGNETISM

BY

ELMER H. WILLIAMS

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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THE ELECTRON THEORY OF MAGNETISM

By ELMER H. WILLIAMS, ASSOCIATE IN PHYSICS

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ELECTRON THEORY OF MAGNETISM

I. Essential Features of the Electron Theory of Magnetism

1. *Introduction.*— During the last decade the development of the subject of magnetism has made rapid strides. Not only have the older theories and methods been extended by improved facilities, but new theories have been advanced which are intended to correlate the great mass of data and facts, and thus enable our present knowledge to be extended along new lines. Among the new theories which have been advanced by various authors, the *electron theory of magnetism* is one of the most important and interesting. This theory seems to account for magnetic phenomena in a very direct way. We have only to assume that the molecular currents of Ampère, which form the elementary magnets, are revolving electrons in order to express Ampère's theory of magnetism in terms of the electron theory. However, a closer study of their orbits, due to Voigt and J. J. Thomson, showed that these currents cannot account sufficiently for the phenomena of diamagnetic and paramagnetic bodies. It was only on the basis of the researches of Curie that Langevin was able to give a more satisfactory theory of diamagnetism and paramagnetism.

The theory, worked out by Langevin for paramagnetic gases only, was later extended by Weiss to ferromagnetic substances. Weiss introduced a new notion into the theory of magnetism, viz., that of an intrinsic or molecular magnetic field by means of which he could account in a very beautiful way for the magnetic properties of the crystal pyrrhotite and many of the magnetic properties of iron, nickel, and cobalt. He has also contributed most essentially to our experimental knowledge of the ferromagnetic phenomena.

One of the co-workers of Weiss in the fundamental investigations on pyrrhotite was J. Kunz, who also contributed to the theory of magnetism by determining the elementary magnetic moment and the charge of the electron from purely magnetic phenomena. In his lectures on the electron theory given at the University of Illinois, Kunz gave an account of the present theory of magnetism and of the experimental and theoretical work of Weiss. The author of this bulletin has used these lectures as a basis, drawing in addition, from the works of the various authors who have made further experimental advances.

Weiss recently advanced a new theory in which the magnetism of a substance appears to be made up of magnetons just as a negative electrical charge is an aggregation of electrons. This theory, if confirmed by further experimental evidence, represents a new fundamental step in the development of our knowledge of the material universe. The experimental evidence, however, seems hardly strong enough to warrant a detailed discussion of it in this bulletin.

The present theory of magnetism as developed by Langevin and Weiss is open to certain objections and fails to explain a considerable number of magnetic phenomena. It gives for instance no connection between the elastic and magnetic phenomena of ferromagnetic substances. It is possible that even the foundation of the present theory will undergo changes, but the experimental facts which have been considered in developing the present form of the theory will still be of value. It seems advisable, therefore, in order to give to this bulletin a more permanent value, to lay considerable stress on the experimental methods applied by Weiss and his followers and to give also the results obtained since the time when the classical book on magnetism was written by Ewing.

The first part of the present bulletin contains, therefore, the essential features of the electron theory of magnetism, the second and the third parts give an account of the properties of ferromagnetic crystals, while the fourth part gives further experimental evidence in favor of the electron theory of magnetism together with an account of some of the phenomena for which the theory in its present form fails to give a satisfactory explanation.

2. *Kinds of Magnetism.*— Bodies are divided, from the point of view of their magnetic properties, into three distinct groups: ferromagnetic, paramagnetic, and diamagnetic.

Under ferromagnetic substances are classed those substances of which the intensity of magnetization at saturation is of the same order of magnitude as that of iron. They are iron, nickel, cobalt, magnetite, pyrrhotite, and the Heusler alloys (which consist of copper, manganese and aluminum).

Paramagnetic substances are those which, while they become magnetized in the direction of the field, do so very feebly. Among paramagnetic bodies are found oxygen, nitrogen dioxide, palladium, platinum, manganese, and the salts of various metals.

Diamagnetic bodies, which include the greater number of all simple and compound bodies, have properties very different from those of either ferromagnetic or paramagnetic bodies. When placed in a magnetic field, they become slightly magnetized in a direction opposite to the direction of the field.

Some bodies, such as iron, when heated, show a gradual transition from the ferromagnetic to the paramagnetic state or vice versa, but as yet no body, with the exception of tin, has been found which, by change of physical conditions, will pass from the diamagnetic to the paramagnetic state.

Within the last decade a large amount of work has been done on the ferromagnetic substances, magnetite, hematite, and pyrrhotite, which are found in nature in crystals of such size and shape as will permit of a study of their magnetic properties. However, it is not possible to obtain comprehensive results with magnetic crystals merely by adapting to them the methods that have been applied with success to isotropic substances.

In the case of isotropic substances, the intensity of magnetization has always the same direction as the field, and, as in all directions the behavior is the same, it is sufficient to apply to the substance a magnetic field of any direction and to determine the intensity of magnetization corresponding to each of its values. It is true that it is necessary to take into account the different secondary phenomena, hysteresis, retentivity, etc., which influence considerably the character of the principal phenomena.

For allotropic substances, it is necessary to consider, besides the magnitude of the field and of the intensity of magnetization, their directions, which, in general, are different for these two quantities. In place of a function of one variable there will be a system of three functions of three variables if one represents the field and the magnetization by their components. As the secondary phenomena are as complicated as in isotropic substances, it is easy to see that the complete investigation of the magnetic properties of a crystal constitutes a very difficult problem.

3. *General Properties of Electrons.*—The electron theory of magnetism supposes that the atom is made up of positive and negative electricity, the latter always occurring as exceedingly small particles called electrons, and that these electrons, whenever they occur, are always of the same size and always carry the same quantity of electricity. It is this peculiar way in which the negative electricity occurs both in the atom and when free from matter that gives to the theory its name. An electron is then, an "atom" of electricity, or the smallest amount of electricity which can be isolated. These electrons are given out by all bodies at a sufficiently high temperature accompanying the phenomena of radiation. These electrical particles leave the metals and other substances under the action of visible and invisible light, Roentgen rays, radium rays, etc. They appear in most of the radioactive processes and in chemical reactions. Thus when a metal is oxidized it emits electrons. Whatever be the source of the electron, its electrical charge has always been found to be from 4.65×10^{-10} to 4.69×10^{-10} absolute electrostatic units. The mass of the electron is about 1,800 times smaller than the mass of the atom of hydrogen, which is the lightest chemical element and which has a mass of about 1.61×10^{-24} grams. The mass of the electron is not ordinary chemical or ponderable matter, but apparent or electromagnetic mass and is due to the electromagnetic field which surrounds the electron in motion. The radius of the electron has been found to be 1.8×10^{-13} cm. To illustrate the size of an electron as compared with an atom, imagine a hydrogen atom increased in volume to that of a large cathedral, the electron being increased proportionally. Then the volume of the electron would be that of a fly flying about in the vast space. In spite of this minute size of the electron, or rather because of this minute size, the actions of the elementary charge are surprisingly great. Thus the electrical field on the surface of the electron is 1.4×10^{15}, or 10^{12} times stronger than any which we are able to produce by artificial means.

An electric current in a metal consists of electrons in motion, while a current through electrolytic solutions and through gases at ordinary and reduced pressures consists of positive and negative ions. As the pressure in a discharge tube becomes smaller, the electrical current is carried more and more by free electrons or cathode rays which, by their impact on a solid obstacle, give rise to Roentgen rays.

The electron theory aims to explain all the phenomena of light, electricity, and magnetism, and in many cases it is the only theory that is able to explain the great variety of physical phenomena. The electron forms a part of each atom of the universe and it plays an important rôle in the chemical theories of matter. It is probable that the forces of affinity in the chemical reactions can be reduced to electrical forces between the electron and the positive charge of the atom. Thus chemical phenomena are drawn into the circle of the electron theory. Even mechanics, the oldest branch of exact natural science, is affected by the discovery that the mass of the electron depends on its velocity, so that Newton's equations of dynamics, the basis of the physical science, have to be slightly changed*. Finally, in the radioactive transformations, in which one element is transformed into another element, the electron plays an essential rôle.

An electron in motion is surrounded by a magnetic field. When an electron moves in a closed orbit, it is accompanied by a permanent magnetic field identical with that of an elementary magnet. Ampère considered the elementary magnets of iron as due to electrical currents flowing in closed molecular orbits without resistance. If we replace these currents of Ampère's theory by electrons moving in closed orbits, we have the fundamental idea of the electron theory of magnetism.

4. *Electromagnetic Force Due to an Electron in Motion.*— Rowland's experiments show that a moving electron is surrounded by a magnetic field. Consider a small element dl, Fig. 1. of the conductor carrying the current i. Let m be the magnetic pole, and ϕ the angle between the direction of the current and the radius r. Then the electromagnetic force produced at m by this element is

$$dK = \frac{midl \sin \phi}{r^2}$$

Suppose

$$m = 1$$

then

$$dK = \frac{idl \sin \phi}{r^2} \quad \text{.......................} (1)$$

Suppose now that the current i is that due to an electron moving with a velocity v, where v is not greater than one third that of the velocity of light. If the electron moves through the distance dl in the time dt, we will have

$$dl = vdt \quad \text{.......................} (2)$$

Also

$$i = \frac{e}{dt} \quad \text{.......................} (3)$$

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* For all engineering purposes and for the motion of the heavenly bodies this change is too small to be considered. It is only when the velocity of the mass approaches in magnitude the velocity of light that the effect is appreciable.

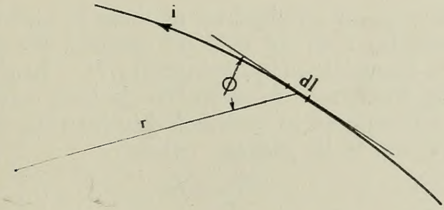

FIG. 1

Substituting (2) and (3) in (1) we get

$$dK=\frac{ev\sin\phi}{r^2} \qquad (4)$$

If we consider a sphere of radius r, Fig. 2, with an electron at the center moving with a velocity v, the magnetic force at the point m will be, from equation (4)

$$dK=\frac{ev\sin\phi}{r^2}$$

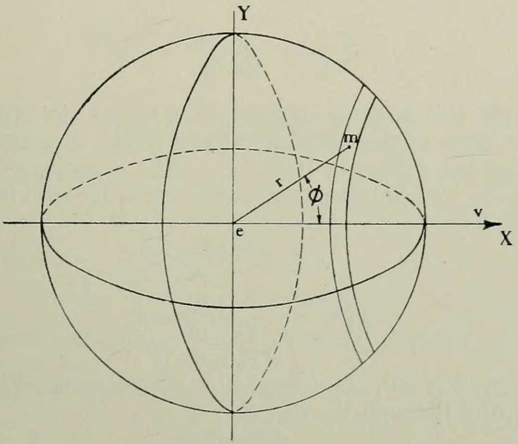

FIG. 2

This is the same for all points on the surface of the sphere where ϕ is the same. Therefore there is a circle around the sphere where the magnetic force is constant. The direction of the magnetic force is at right angles to the motion of e, or in a plane perpendicular to the x-axis.

5. *Diamagnetism.*— As defined before, a diamagnetic body is one which when placed in a magnetic field becomes slightly magnetized in a direction opposite to that of paramagnetic substances. Thus, a cylindrical diamagnetic body will set itself perpendicular to a magnetic field.

Consider in any atom an electron of mass m and charge e, Fig. 3. moving with a velocity v, in an orbit of radius r the plane of which is perpendicular to a magnetic field of intensity H. In the absence of the magnetic field, the centrifugal force on the electron is opposed by elastic forces, which we will suppose to be directed toward the center of the orbit and to be proportional to its radius. Then

$$\frac{mv^2}{r}=fr \qquad (5)$$

where f is the force of attraction toward the center, when

$$r=1\ cm.$$

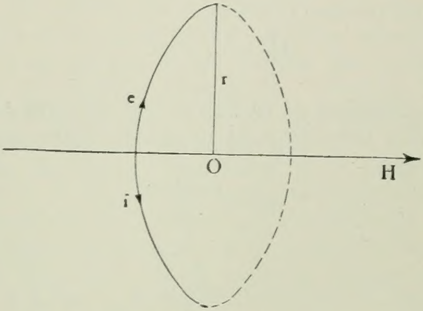

FIG. 3

Now apply the external magnetic field, and the electron is subject to a force at right angles to the field and to the direction of its motion, that is, along the radius of its orbit. The magnitude of this force is found as above by applying the fundamental law of electromagnetism. As before we have

$$dK=\frac{mi\ dl\sin\phi}{r^2}$$

$$=\frac{m}{r^2}idt\frac{dl}{dt}\sin\phi$$

$$=Hev\sin\phi$$

If the angle $\phi = 90°$, the force which the magnetic field exerts on the electron moving through the distance dl is

$$dK=Hev$$

Applying Ampère's rule we see that if the electron is negatively charged the force is directed outward along the radius. Since the electromagnetic forces acting on the electron are perpendicular to the direction of its motion, the magnitude of its velocity v is unchanged by the action of these forces.

Denoting the period of the new orbit produced when the field H is acting by T' and its radius by r', we have

$$\frac{mv^2}{r'}=fr'-Hev$$

or

$$\frac{mv^2}{r'^2}=f-\frac{Hev}{r'} \qquad (6)$$

(6) − (5) gives

$$\frac{mv^2}{r'^2}-\frac{mv^2}{r^2}=-\frac{Hev}{r'} \quad \text{.......................(7)}$$

$$T=\frac{2\pi r}{v}$$

or

$$T^2=\frac{4\pi^2r^2}{v^2}$$

Therefore

$$\frac{v^2}{r^2}=\frac{4\pi^2}{T^2} \quad \text{.......................(8)}$$

Also

$$T'=\frac{2\pi r'}{v}$$

or

$$T'^2=\frac{4\pi^2r'^2}{v^2}$$

Therefore

$$\frac{v^2}{r'^2}=\frac{4\pi^2}{T'^2} \quad \text{.......................(9)}$$

Substituting (8) and (9) in (7)

$$\frac{4\pi^2\, m}{T'^2}-\frac{4\pi^2\, m}{T^2}=-\frac{2\pi He}{T'}$$

whence

$$\frac{1}{T'^2}-\frac{1}{T^2}=-\frac{He}{2\pi mT'}$$

or

$$\frac{T^2-T'^2}{T^2T'^2}=-\frac{He}{2\pi mT'}$$

This may be written

$$\frac{(T+T')(T-T')}{T^2T'^2}=-\frac{He}{2\pi mT'} \quad \text{...............(10)}$$

Now the diamagnetic phenomena are very small, therefore we can without appreciable error put

$$T+T'=2T$$

and

$$TT'=T^2$$

Substituting these values in (10) we have,

$$T-T'=-\frac{HeT^2}{4\pi m} \quad \text{....................(11)}$$

In order to calculate the intensity of the induced magnetization, let us replace the revolving electron by an equivalent current flowing in a circuit coincident with its orbit. The strength i of the equivalent current is given by

$$i=\frac{e}{T}$$

Now the magnetic moment of a circuit of area A carrying a current i is given by

$$M_1=Ai$$

In the case of an undisturbed electron revolving in an atom

$$i=\frac{e}{T}$$

and $$A=\pi r^2$$

where r is the radius of the orbit.

Therefore the moment of the equivalent elementary magnet is

$$M_1=\frac{e}{T}\pi r^2$$

When the magnetic field H is applied this becomes

$$M_1'=\frac{e}{T}\pi r'^2$$

Let

$$M_1-M_1'=\triangle M_1$$

be the induced magnetic moment for one revolving electron. Therefore

$$\triangle M_1=e\pi\left(\frac{r^2}{T}-\frac{r'^2}{T'}\right).$$

If we have N electrons revolving in unit volume, the induced magnetic moment per unit volume is

$$\triangle M=N\triangle M_1=Ne\pi\left(\frac{r^2}{T}-\frac{r'^2}{T'}\right)\dots\dots\dots\dots(12)$$

From (8)

$$\frac{r^2}{T}=\frac{v^2T}{4\pi^2}\dots\dots\dots\dots(13)$$

From (9)

$$\frac{r'^2}{T'}=\frac{v^2T'}{4\pi^2}\dots\dots\dots\dots(14)$$

Substitute (13) and (14) in (12) and

$$\triangle M=\frac{Ne\pi v^2}{4\pi^2}(T-T')\dots\dots\dots\dots(15)$$

From (11)

$$T-T'=-\frac{HeT^2}{4\pi m}$$

Therefore (15) becomes

$$\triangle M=-\frac{Ne^2v^2HT^2}{(4\pi)^2m}\dots\dots\dots\dots(16)$$

Substitute (13) in (16) and there is obtained

$$\triangle M=-\frac{Ne^2Hr^2}{4m}$$
$$=-\frac{HNe^2r^2}{4m}$$

This is the induced magnetism per unit volume due to an external magnetic field H. Therefore $\triangle M$ is proportional to H, or

$$\triangle M=-kH,$$

where $$k=\frac{Ne^2r^2}{4m}$$

is defined to be the diamagnetic susceptibility.

All the quantities on the right hand side of this equation are essentially positive, hence ΔM is negative and the body is diamagnetic whatever the sign of the electronic charge e. Thus all substances possess the diamagnetic property according to the above theory. Some substances are also paramagnetic, that is, one phenomen on is superimposed on the other.

In the above calculations it has been assumed that all the electronic orbits are so arranged that their axes are in the direction of the magnetic intensity of the inducing field and their planes perpendicular to their direction. It would be more accurate to assume that the axes are distributed in all directions. However, the change introduced by this assumption would consist only in multiplying the right hand side of the last equation by a proper fraction whose value is not very different from unity.

Multiplying numerator and denominator of the above expression for k by m, the mass of an electron, then

$$k = \frac{Nme^2r^2}{4m^2}$$

$$= \frac{\rho}{4}\left(\frac{e}{m}\right)^2 r^2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

where ρ is the density of the electrons, or the mass of the electrons per unit volume. It is seen that the effect produced contains as one factor, the square of the ratio $\frac{e}{m}$, and since this ratio is at least a thousand times greater for the negative electrons than for the positive corpuscles it is the former, which are present in all substances, that play the essential rôle in the production of diamagnetism.

In the case of water, of which the diamagnetic constant is $.8 \times 10^{-6}$, the density ρ of the negative electrons, which constitute only a part of the molecule, is less than unity and probably greater than 1/2000, the ratio of the mass of a negative electron to that of the atom of hydrogen. The ratio e/m is known and is

$$\frac{e}{m} = 1.8 \times 10^7$$

absolute electromagnetic units.

Substituting the above values in equation (17) we get, using 1 for ρ,

$$r < 10^{-8}\ cm.$$

Substituting 1/2000 for ρ we get

$$r > 2 \times 10^{-10}\ cm.$$

Therefore

$$2 \times 10^{-10} < r < 10^{-8}.$$

Experimental determinations indicate that r actually lies within these limits.

As the electronic orbits are considered to belong to the interior of the atom, which is not affected by temperature, we should expect that the diamagnetic susceptibility does not depend upon temperature. Curie's results indicate that, in general, this is the case, although du Bois and Honda found a large number of exceptions. Also the position of the lines of the spectrum, which are due to the revolution of electrons inside the atom, is almost entirely independent of the temperature.

Equation (16) above may be interpreted simply as follows. The equation

$$\triangle M = -\frac{HNe^2r^2}{4m}$$

expresses the increase in the magnetic moment per unit volume, which contains N electrons. For one electron the increase in the magnetic moment is

$$\triangle M_1 = -\frac{He^2r^2}{4m}$$
$$= -\frac{He^2r^2\pi}{4m\pi}$$

This may be written

$$\triangle M_1 = -\frac{He^2A}{4\pi m} = -\frac{e^2}{4\pi m}HA \quad \ldots\ldots\ldots\ldots\ldots (18)$$

if the electron describes a circular orbit whose area is πr^2. The change of the magnetic moment of the electronic orbit is determined by the flux of magnetic induction HA which is produced by the external field passing through the orbit.

This is exactly the result that is obtained by merely applying to the electronic orbits the elementary laws of induction for the elementary circuits. Let us suppose that the resistance of the orbit is zero, and the self-inductance equal to $\triangle L$. If i is the current, the equation for the induced e. m. f. gives

$$\frac{d(\triangle Li)}{dt} = -\frac{d(\phi)}{dt} = -\frac{d(HA)}{dt}$$

whence

$$\triangle Li = -HA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

if H is zero in the beginning.

$$\triangle M_1 = \triangle Ai = -\frac{e^2}{4\pi m}HA$$

Substituting the value of HA given by (19),

$$\triangle Ai = \frac{e^2}{4\pi m}\triangle Li$$

It is sufficient to take

$$Ai = \frac{e^2}{4\pi m}Li$$

Whence since

$$A = \pi r^2$$
$$L = \frac{4\pi^2mr^2}{e^2}$$

Thus the apparent self-induction is proportional to the mass of the electron and to the square of its radius, and inversely proportional to the square of the charge. This self-induction will identify itself with a real self-induction corresponding to the creation of a magnetic field by the electronic orbit, only if the inertia of the electron is wholly of electromagnetic origin.

If we assume that the self-induction of the current is due to the electromagnetic inertia of the electron,

$$\tfrac{1}{2}Li^2 = \tfrac{1}{2}mv^2$$

where

$$i = \frac{e}{T},$$

and

$$v = \frac{2\pi r}{T}$$

Whence

$$L\frac{e^2}{T^2} = \frac{m\,4\pi^2 r^2}{T^2}$$

or

$$L = \frac{4\pi^2 m r^2}{e^2}$$

which corresponds to the value of L found above.

The diamagnetic modification corresponds to a slight change in the magnetic moment of the original circuit. We have seen that this magnetic moment has the value

$$Mi = Ai = A\frac{e}{T}.$$

This magnetic moment, owing to the variation of the magnetic field, undergoes a change

$$\triangle M_1 = -\frac{He^2r^2}{4m}$$

The relative variation of the magnetic moment is

$$\frac{\triangle M_1}{M_1} = -\frac{HTe}{4\pi m}$$

which in the case of all diamagnetic bodies is very small. Now $\frac{e}{m}$ is of the order of 10^7 for negative electrons and still less for positive particles. T must necessarily have a value larger than that of the period of light which is of the order of 10^{-15}, for if the time of vibration were so small, the magnet would be a spontaneous source of light and permanent magnetism would be impossible. Let us assume that T is of the order 10^{-12}, which corresponds to the longest wave length that has been isolated in the spectrum of mercury vapor. Then in order to make $\frac{\triangle M_1}{M_1}$ approach unity, H needs to be of the order of only 10^6. Now we are able to produce fields of the order of 10^5, which, on the above assumption, would cause a change in the magnetic moment of the diamagnetic substance of one tenth of that of its original value.

6. *Magnetic Energy.*— In order to calculate the energy required to produce the diamagnetic modification, let us assume an electronic orbit which is without motion during the establishment of the magnetic field H. The magnetic force is perpendicular to the plane of the orbit and therefore produces no work. If there is a displacement dl along the

orbit (Fig. 4) and the electric force is E, the work done is

$$\begin{aligned} dW_1 &= eEdl \cos \alpha \\ &= e\,(E_x dx + E_y dy + E_z dz) \end{aligned}$$

or

$$W_1 = e\int_C (E_x dx + E_y dy + E_z dz).$$

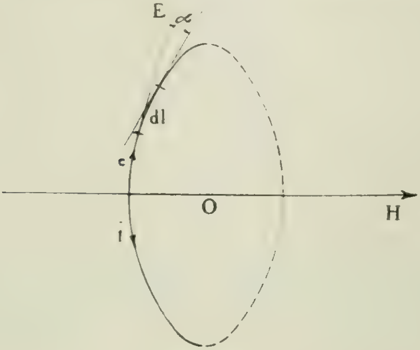

FIG. 4

During the time T of one revolution of the electron, which is of the order of 10^{-15} seconds, a time extremely short in comparison with the time necessary for the establishment of the field by the creation of currents or the displacement of magnets, E_x, E_y, E_z will not change appreciably and the work W may be calculated by an application of Stokes's Theorem.

$$W_1 = e\int_C (E_x dx + E_y dy + E_z dz) = e\iint_A \Big[\Big(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\Big)\cos nx$$

$$+ \Big(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\Big)\cos ny + \Big(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\Big)\cos nz\Big]dA \quad \ldots\ldots\ldots (20)$$

where n is the normal to the surface.

Introduce Maxwell's equations,

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial H_x}{\partial t} &= -\Big(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\Big) \\ \frac{1}{c}\frac{\partial H_y}{\partial t} &= -\Big(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\Big) \\ \frac{1}{c}\frac{\partial H_z}{\partial t} &= -\Big(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\Big) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

c, the ratio between electrical energy expressed in electrostatic and electromagnetic units, is already included in E_x, E_y, and E_z of equation (20), therefore it should be omitted from equation (21) before substitution. Substituting (21) in (20), we have

$$W_1 = -e\iint_A \Big[\frac{\partial H_x}{\partial t}\cos nx + \frac{\partial H_y}{\partial t}\cos ny + \frac{\partial H_z}{\partial t}\cos nz\Big]dA$$

Now

$$H_x\cos nx+H_y\cos ny+H_z\cos nz=H$$

Therefore

$$W_1=-e\int\!\!\int_A\frac{dH}{dt}\,dA$$

$$=-e\frac{dH}{dt}A$$

since $\frac{dH}{dt}$ is the same for all points of the area A.

The work done by the increasing magnetic field per unit of time is

$$W_1=-\frac{e}{T}\,\frac{dH}{dt}A$$

$$=-\,iA\,\frac{dH}{dt}$$

Therefore

$$W_1=-\,M_1\frac{dH}{dt}$$

The work done in the time dt is

$$dW_1=\;-\,M_1\,dH$$

whence

$$W_1=-\int_o^H M_1dH\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

where M_1 is the magnetic moment of the electronic current in the direction dH. This work, done during the establishment of the magnetic field H by the current, or by the displacement of a magnet, is transformed into kinetic or potential energy of the electron which produces the electronic current. It represents the potential energy between the revolving electron and the magnetic field H. This transformation of energy accompanies the production of the diamagnetic state.

If the initial magnetic moment of the revolving electron is equal to M_o, the external field being zero, the magnetic moment under the action of the field will be

$$M_o+\triangle M_1=M_o-\;\frac{e^2r^2H}{4m}$$

From (22)

$$W_1=-\int_o^H\left(M_o-\frac{e^2r^2H}{4m}\right)dH=-\,M_oH+\frac{e^2r^2H^2}{8m}$$

If the molecule has N orbits, the resultant initial moment M will be

$$M=M_oN$$

whence

$$W=-\,MH+\frac{e^2r^2H^2N}{8m}$$

If the body is purely diamagnetic, M is zero and we have simply

$$W=\frac{e^2r^2H^2}{8m}\,N$$

This is the energy brought into play in purely diamagnetic phenomena. It is always present even in the case where the body is paramagnetic, but is small in comparison with the energy of the latter.

7. *Paramagnetism.*— We have seen that in all cases the creation of an exterior magnetic field modifies the electronic orbits by polarizing diamagnetically, all the molecules. This phenomenon is manifested only in the case where the resultant moment of the electronic orbits is zero, when the matter is diamagnetic in the ordinary sense of the word.

If the resultant moment is not zero, upon the diamagnetic phenomena is superimposed another phenomenon due to the orientation of the molecular magnets by the external field. The substance is then paramagnetic if the mutual action between molecular magnets is negligible, as in the case of gases and of solutions, and ferromagnetic in the case where the mutual actions play the essential rôle. As soon as the paramagnetism appears it is, as a rule, enormous in comparison with the diamagnetism and therefore completely conceals it. This explains the absence of continuity between paramagnetism and diamagnetism; paramagnetism may not exist; but if it exists, it hides completely the diamagnetism.

Therefore, substances whose atoms have their electrons in revolution in such a way that their effects are additive, are paramagnetic. The atoms of such substances may be looked upon as elementary magnets. The energy of such an elementary magnet may be represented by

$$W = -MH \cos \alpha$$

where α is the angle that the magnet makes with the magnetic field H.

If a magnetic field acts on a paramagnetic substance in a gaseous state and if the molecules have no thermal agitation, they will rearrange themselves in a direction parallel to the magnetic field. By virtue of this rearrangement they will, like a falling body, lose their original potential energy and acquire kinetic energy.

8. *Curie's Rule.*— Let us consider a paramagnetic body in the gaseous state, such as oxygen, whose molecules have a magnetic moment M. The molecules of such a body will turn, when under the influence of a uniform magnetic field H, in such a way as to place their magnetic axes parallel to the field. Let us calculate the magnetic moment per unit volume after the rearrangement has taken place.

If the magnetic moment makes an angle α with the direction of the uniform field H, then the molecule possesses a potential energy equal to

$$-MH \cos \alpha$$

The increase of this potential energy is derived from the kinetic energy of rotation of the molecules in the same way in which the potential energy of gravitation of the molecules of a gas is derived from the kinetic energy of translation when it is rising. The resultant inequalities in the distribution of kinetic energy between the orientations and the degrees of freedom of the molecules, rotation and translation, are not compatible with thermal equilibrium. A rearrangement takes place at the instant of the collisions during which the magnetic polarity appears and the energy

$$-HdM$$

of thermal agitation turns into potential energy of magnetization.

If the molecules have no relative potential energy of orientations, as in the case of gases and liquids, it is necessary, in order to maintain the medium at a constant temperature during the process of magnetization, to introduce into the system the energy $-HdM$, in the form of heat. In the case of a solid, where the molecules have a potential energy of orientation, this heat will be introduced only in case a cycle has been performed.

In the former case, we may apply the principles of thermodynamics in order to find the law experimentally established by Curie, since the process is perfectly reversible. The magnetic moment M for a given mass of the substance will be a function of the magnetic field H, and of the absolute temperature T. During a reversible change dH, dT, one can take out of the system a quantity of heat of which the portion which depends on H is

$$\begin{aligned} dQ &= HdM \\ &= H\left(\frac{\partial M}{\partial H}\,dH + \frac{\partial M}{\partial T}\,dT\right). \end{aligned}$$

In a reversible process

$$dA = \frac{dQ}{T}$$

Therefore

$$dA = \frac{H}{T}\frac{\partial M}{\partial T}\,dT + \frac{H}{T}\frac{\partial M}{\partial H}\,dH.$$

According to the second law of thermodynamics this must be an exact differential, therefore

$$\frac{\partial}{H\partial}\left(\frac{H}{T}\frac{\partial M}{\partial T}\right) = \frac{\partial}{\partial T}\left(\frac{H}{T}\frac{\partial M}{\partial H}\right)$$

or

$$\frac{1}{T}\frac{\partial M}{\partial T} + \frac{H}{T}\frac{\partial^2 M}{\partial T\partial H} = \frac{H}{T}\frac{\partial^2 M}{\partial H\partial T} - \frac{H}{T}\frac{2\partial M}{\partial H}$$

whence

$$\frac{1}{T}\frac{\partial M}{\partial T} = -\frac{H}{T^2}\frac{\partial M}{\partial H}.$$

Therefore

$$\frac{\partial M}{\partial T} = -\frac{H}{T}\frac{\partial M}{\partial T}$$

or

$$\frac{\dfrac{\partial M}{\partial T}}{\dfrac{\partial M}{\partial H}} = -\frac{H}{T} \qquad \text{(23)}$$

The general integral of (23) is

$$M = f(H/T) \qquad \text{(24)}$$

In the beginning of the magnetization, where the susceptibility may be considered constant at a given temperature

$$M = kH \qquad \text{(25)}$$

Comparing (24) with (25), we see that k, the paramagnetic susceptibility, must vary inversely as the absolute temperature, that is

$$k = A/T$$

This is called Curie's Rule, and the constant A is sometimes called Curie's constant. When this rule was first given out by Curie, it was thought to be general, but since then some substances have been found in which temperature does not affect the diamagnetic susceptibility k, and others in which k actually increases with increase of temperature.

9. *Langevin's Theory.*— The following comparison, which is due to Langevin,* will make clear the theory which precedes. Imagine a gaseous mass contained in a given receptacle Fig. 5, without being subject to the action of gravity. The molecules will distribute themselves in such a manner that the density of the gas will be the same at all points, which is similar to that which takes place in the case of a magnetic gas, such as oxygen, in the absence of an exterior magnetic field when the molecules have their axes distributed uniformly in all directions, Fig. 7.

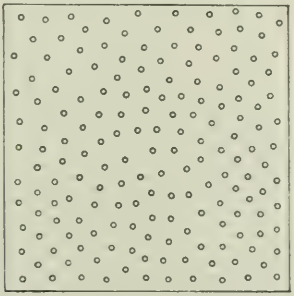

FIG. 5

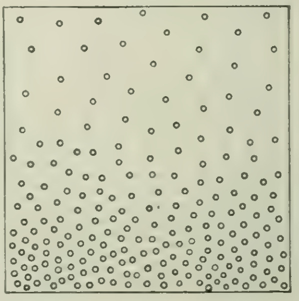

FIG. 6

If the force of gravitation is applied, Fig. 6, the molecules will acquire an acceleration directed toward the base, and, in the absence of mutual collisions, each molecule will have a greater velocity at the bottom than at the top of the vessel. But this inequality of velocity is incompatible with thermal equilibrium, and a rearrangement will take place due to the mutual collisions, after which the distribution which is established is given by the formula of barometric pressure. The center of gravity is lowered, and in order to maintain the gas at the initial temperature, it is necessary to remove from it a quantity of heat equivalent to the product of the mass of the gas by this lowering of the center of gravity, or equivalent to the loss of potential energy. One deduces from a thermodynamic reasoning analogous to that given above, that this lowering of the center of gravity is inversely proportional to the absolute temperature.

After the rearrangement in a mass of gas of uniform temperature, the distribution of the molecules takes place between the various regions in a manner such that the molecules will be more numerous where the potential energy is the least, that is to say, at the lowest points in the case of gravity. Boltzmann has calculated the distribution by generalizing the law of barometric pressure. The ratio of the densities of the gas in two points between which the potential energy varies by W is

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*P. Langevin, Ann. de Chem. et de Phys., Ser. 8, Vol. 5, p. 70, 1908.

$$e^{\frac{W}{RT}}$$

where e is the base of Naperian logarithms, T the absolute temperature of the gas and R the constant of the equation of a perfect gas, a constant such that, according to the kinetic theory, RT represents two thirds of the mean kinetic energy of translation.

The change when the magnetic field H is applied to a paramagnetic gas such as oxygen, Figs. 7 and 8, is the same as in the case of gravity except that here we have a rotation of the axes of the elementary magnets which assume the direction of H. Here too there is a loss of potential energy and a gain of heat, the rise in temperature being due to the thermal agitation of the molecules which produce a certain amount of heat due to their rotation.

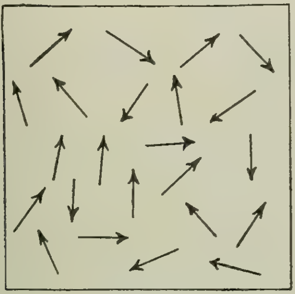

FIG. 7

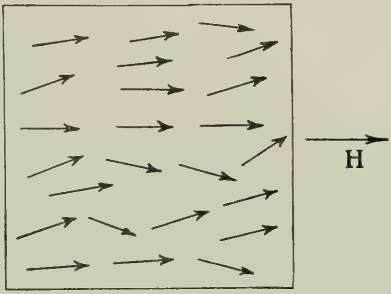

FIG. 8

The distribution of the molecules between the various orientations will be determined by the static equilibrium which will establish itself under the superimposed influence of the potential magnetic energy, $MH \cos \alpha$, and the energy RT of thermal agitation, the molecules being from preference oriented in the direction of least potential energy, that is to say, with their magnetic axes in the direction of the field. If one considers the distribution of the magnetic axes between the various directions, the density per unit of solid angle will vary from one direction to the other proportional to

$$e^{\frac{mH \cos \alpha}{RT}}$$

all directions being equally probable if M or $H=O$. The number of molecules whose axes are directed within the solid angle $d\alpha$, Fig. 9, will be

$$dn = Ke^{\frac{MH \cos \alpha}{RT}} d\omega$$

where the element $d\omega$ is a zone of aperture $d\alpha$ around the direction of the field

$$d\omega = 2\pi \sin \alpha d\alpha$$

α varying from 0 to π.

Therefore

$$dn = Ke^{\frac{MH\cos\alpha}{RT}} 2\pi \sin\alpha d\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

The total number of molecules per unit volume N will be

$$N = 2\pi K \int_{o}^{\pi} e^{a\cos\alpha} \sin\alpha d\alpha$$

where

$$a = \frac{MH}{RT}$$

Let

$$\cos\alpha = x$$

then

$$dx = -\sin\alpha d\alpha$$

and

$$N = 2\pi K \int_{-1}^{+1} e^{ax} dx$$

$$= \frac{2\pi K}{a} \Big[e^{ax} \Big]_{-1}^{+1}$$

$$= \frac{2\pi K}{a} \left(e^{a} - e^{-a} \right)$$

Now

$$\frac{e^{a} - e^{-a}}{2} = \sinh a$$

Therefore

$$N = \frac{4\pi K}{a} \sinh a$$

whence

$$K = \frac{aN}{4\pi \sinh a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The total magnetic moment of the N molecules is evidently directed parallel to the field and is equal to the sum of the projections of the component moments on this direction. For the unit of volume supposed to contain N molecules, this resultant moment represents the intensity of magnetization I.

$$I = \int M \cos\alpha \, dn.$$

Substitute for dn its value given by equation (26) and

$$I = \int_{o}^{\pi} M \cos\alpha K e^{a\cos\alpha} 2\pi \sin\alpha \, d\alpha$$

Therefore

$$I = 2\pi MK \int_{-1}^{+1} x e^{ax} dx$$

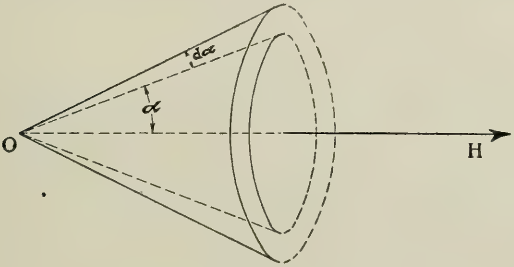

FIG. 9

Now

$$\int_{-1}^{+1} xe^{ax}\,dx = 2\left(\frac{\cosh a}{a} - \frac{\sinh a}{a^2}\right),$$

therefore

$$I = 4\pi MK\left(\frac{\cosh a}{a} - \frac{\sinh a}{a^2}\right)$$

Substitute the value of K given by equation (27) and

$$I = MN\left(\frac{\cosh a}{\sinh a} - \frac{1}{a}\right)$$

For a given number of molecules N, I is therefore a function solely of a, that is, of H/T, in accordance with the results given by thermodynamics.

The expression $\left(\frac{\cosh a}{\sinh a} - \frac{1}{a}\right)$ vanishes with a, which is proportional to H, and tends toward unity when a increases indefinitely, the intensity of magnetization approaching the maximum value $I_m = MH$ which corresponds to saturation, that is, the condition when all the molecular magnets are oriented parallel to the magnetic field.

The curve of magnetization of a magnetic gas at constant temperature ODE, Fig. 10, representing I/I_m as a function of a, that is, as a function of H, would be represented by the expression

$$I/I_m = \frac{\cosh a}{\sinh a} - \frac{1}{a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$
$$= f(H/T).$$

It is evident that the magnetic susceptibility will not be constant and I will be proportional to H only for values small compared with unity.

Developing equation (28) in series we have

$$\frac{I}{I_m} = \frac{\cosh a}{\sinh a} - \frac{1}{a} = \frac{1}{3} - \frac{2}{90}a^3 + \frac{4}{45.42}a^5 \ldots\ldots\ldots\ldots (29)$$

Taking account only of the terms of the first degree with respect to a we find

$$I = \frac{I_m}{3}a = \frac{M^2NH}{3RT} = k'\frac{H}{T} = kH$$

where

$$k = \frac{M^2N}{3RT}$$

that is the paramagnetic susceptibility is inversely proportional to the absolute temperature, which agrees with the rule obtained experimentally by M. Curie.

If all the molecules were oriented parallel to the field, that is if the body were magnetized to the point of saturation, the intensity of magnetization would be,

$$I_m = MN$$

Combining this with the expression above, we get

$$k = \frac{M^2N^2}{3RTN} = \frac{I^2_m}{3p}$$

p being the pressure of the gas at which k is measured.

Curie found for oxygen at the standard pressure 10^6 and at a temperature of 0° *C.*,

$$k = 1.43 \times 10^{-7}$$

per unit of volume.

$$\begin{aligned} I_m{}^2 &= k \times 3p \\ &= 1.43 \times 10^{-7} \times 3 \times 10^6 \\ &= 0.43 \end{aligned}$$

whence

$$I_m = 0.65$$

This would correspond for liquid oxygen, which is at least 500 times more dense than the gas, to a maximum magnetization.

$$I_m = 325$$

which is not very much smaller than that of iron. In fact, liquid oxygen possesses such intense magnetic properties that it forms a liquid bridge between the poles of an electromagnet.

From the above, one is able to obtain the order of magnitude of the quantity a under ordinary experimental conditions.

$$a = \frac{MH}{RT} = \frac{MNH}{NRT} = \frac{I_mH}{NRT}$$

But NR is the constant of a perfect gas for the unit of volume which is supposed to contain N molecules. Under the normal conditions for which I_m has been calculated, one will have

$$NRT = p = 10^6 \textit{c. g. s. units.}$$

Therefore

$$a = 0.65 \times 10^{-6}H.$$

For a field of 10,000 units a will be 0.0065, and one would still be near the origin of the curve of magnetization, Fig. 10, where the curve coincides with the straight line. In order to make $a = 1$, the region where the curve commences to leave the straight line, it would be necessary to have fields greater than 10^6, which we are not able to produce.

One sees then, in the ferromagnetic substances, the importance of the mutual actions between molecules which alone makes possible magnetic saturation. For the same exterior fields, magnetic saturation is still far removed in the case of paramagnetic substances where mutual actions are not appreciable.

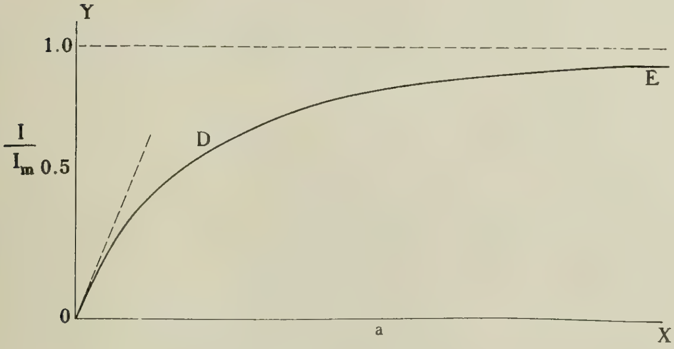

Fig. 10

From this point of view, Curie's comparison of the transition between paramagnetism and ferromagnetism to the transition between gaseous and liquid states where the mutual actions play an essential rôle, is perfectly justifiable. In the pure gaseous state, as in paramagnetism, each molecule reacts individually by its own kinetic energy against the exterior forces of pressure and magnetic field.

10. *Electronic Orbit.*— If one assumes that the magnetic moment M of one oxygen molecule is due to only one electron of charge equal to that of the atom of hydrogen obtained in electrolysis, moving along a circular orbit whose radius is equal to the radius of the molecule of air, 1.5×10^{-8} cm., one can calculate the velocity of the electron along the orbit. We have

$$\begin{aligned} M &= Ai \\ &= A\frac{e}{T} \\ &= \frac{\pi r^2 ev}{2\pi r} \\ &= \frac{rev}{2}. \end{aligned}$$

Also

$$I_m = MN = Ne\frac{rv}{2}.$$

But the product Ne is given by electrolysis, since e is the charge of the atom of hydrogen. Under normal conditions

$$\begin{aligned} Ne &= 1.22 \times 10^{10} \text{ electrostatic units} \\ &= 0.4 \text{ electromagnetic units.} \end{aligned}$$

Therefore

$$I_m = 0.65 = 0.4\frac{1.5 \times 10^{-8}}{2}v$$

whence

$$v = 2 \times 10^8 \textit{ cm. per sec.}$$

Suppose the revolving negative charge e, Fig. 11, to be attracted toward the center of its orbit by an equal positive charge. Then the centrifugal force equals the centripetal force, that is

$$\frac{mv^2}{r}=\frac{e^2}{r^2}$$

or

$$v^2=\frac{e^2}{mr}$$

where e is measured in electrostatic units.

$$e = 4.65 \times 10^{-10}$$
$$r = 1.5 \times 10^{-8} cm.$$
$$m = 8.77 \times 10^{-28}\ gram.$$

Therefore

$$v^2=\frac{(4.65 \times 10^{-10})^2}{8.77 \times 10^{-28} \times 1.5 \times 10^{-8}}$$
$$= 1.6 \times 10^{16}$$

or $$v = 1.2 \times 10^8\ cm.\ per\ sec.,$$

which is in accordance with the value found by the above method.

It is a remarkable thing that the magnetic moment of the molecule of oxygen can be due to the revolution of a single electron. The same is probably true in the case of iron of which the maximum magnetization is, as we have seen, of the same order of magnitude as that of oxygen. The other electrons in the atom neutralize each other as in the purely diamagnetic body.

The paramagnetic electron also probably plays a part in chemical actions, the number of electrons of a molecule acting being equal to the valence. Therefore, the paramagnetic properties of an element change with change of molecular combinations, while diamagnetism seems to be an internal and invariable property of the atom.

11. *Molecular Field of Ferromagnetic Substances.*— Weiss Hypothesis: Each magnetic molecule in a ferromagnetic substance is subject to a uniform intrinsic magnetic field NI proportional to the intensity of magnetization I and acting in the same direction. This molecular magnetic field is due to the action of the neighboring molecules. It may be called the internal magnetic field in comparison with the internal pressure of Von der Waal's equation. This field, added to the external field, accounts for the great intensity of magnetization of ferromagnetic bodies by means of the laws of paramagnetic bodies, in the same way as the internal pressure, added to the external pressure, accounts for the great density of the liquids by invoking the compressibility of the gas.

This hypothesis has proved to be in agreement with experimental facts in a large number of cases. Among these cases may be named the properties of pyrrhotite, the heat developed when a substance passes from the paramagnetic to the ferromagnetic state and the law of temperature and intensity of magnetization of magnetite. The properties of hysteresis of iron can be given a theoretical interpretation and from $H_m = NI$, where N is a constant, I the intensity of magnetization and H_m the intrinsic molecular field, can be calculated the absolute values of the elementary magnets and the fundamental quantities of nature.

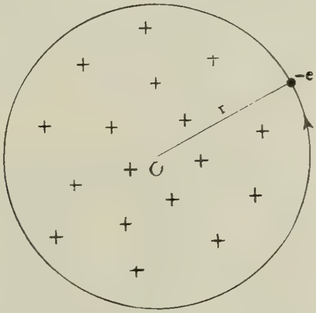

Fig. 11

There are, however, exceptions to this hypothesis, such as the law of approach to saturation and many temperature variations of ferromagnetism. Liquid oxygen does not follow the temperature law nor do iron and nickel at very low temperatures. Furthermore, when iron loses its magnetism we should expect that its volume would increase since the molecules lose their mutual actions and should go farther apart. On the contrary, iron contracts in passing from the ferromagnetic to the paramagnetic state. Colby* has shown that, in the case of nickel, the coefficient of expansion decreases very decidedly as the metal passes through the critical temperature.

II. Experimental Determination of the Magnetic Properties of Crystals

12. *Pyrrhotite.*— Pyrrhotite is one of the most simple of the crystalline magnetic substances. It is composed of sulphur and iron in quantities that are not perfectly constant; it also contains traces of nickel. Various authors have proposed the formulæ Fe_7S_8, Fe_8S_9, etc., all of which can be expressed in the form, n (FeS) + FeS_2. The color of pyrrhotite varies from brown to dark bronze.

Weiss† has shown that, in the case of pyrrhotite, if it be brought into a magnetic field in such a manner that the plane of its hexagonal base is perpendicular to the lines of force, little or no attraction will be exerted on it, while there is a very strong attraction with it in any other position. From this we see that there is one direction in which the magnetization is practically impossible. However, pyrrhotite is strongly magnetized in a plane perpendicular to this direction. This last named plane is called the plane of magnetization. In this plane, which is parallel to the base, there are two directions at right angles to each other in which the magnetization is different: (1) the direction of easy magnetization; (2) the direction of difficult magnetization.

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*W. F. Colby, Phys. Rev., Vol. XXX, No. 4, p. 506, 1910.

†Weiss, Jour. de Phys., ser. 3, Vol. 8, p. 542, 1899.

13. *Apparatus and Methods for Determining the Magnetic Properties of Pyrrhotite.*—Weiss,[*] in his experimental determination of the magnetic properties of pyrrhotite, used two methods: (1) the ballistic method, in which a ballistic galvanometer was used to measure the quantity of electricity induced in a coil surrounding the sample by variations in the magnetic field; (2) a method in which he measured the couple produced on the crystal by the field.

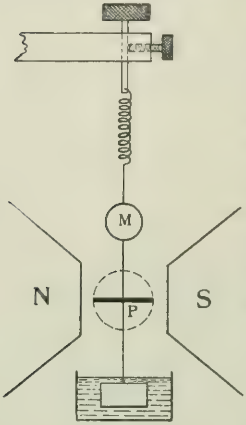

FIG. 12

The latter method is the more sensitive and permits of the examination of much smaller specimens. In this method a disk of pyrrhotite, *P*, Fig. 12, from 1 cm. to 2 cm. in diameter and about ½ mm. thick, is placed between the poles of a magnet of field strength *H*, and the magnetic properties of the crystal are studied with the disk in various positions. The intensity of magnetization is not necessarily in the direction of H, in fact, it is in this direction only in one position of the disk, i.e., when the direction in which the disk is most easily magnetized coincides with the direction of *H*. In general, the intensity of magnetization makes an angle α with *H*.

14. *Component of Magnetization Perpendicular to the Field.*—Suppose the crystal is turned so that the elementary magnet, with center at *O*, Fig. 13, makes an angle α with the direction of the field, *H*.

Let N_e be the moment of the couple acting on the elementary magnet. Then

$$\begin{aligned} N_e &= 2lm\ H \sin \alpha \\ &= \mu\ H \sin \alpha \end{aligned}$$

where $2l$ is the length of the magnet, m its pole strength and μ the magnetic moment of the elementary magnet.

I, the intensity of magnetization, is equal to $n\mu$, where n is the number of elementary magnets per unit volume. Therefore,

$$\begin{aligned} N &= n\mu H \sin \alpha \\ &= I\ H \sin \alpha \end{aligned}$$

where N is the moment of the couple acting on the crystal.

Now, $I \sin \alpha$ equals the component of the intensity of magnetization perpendicular to H. Therefore

$$N = HI_{(p)} = C\alpha'$$

where C is the constant of the apparatus and α' is the deflection of the mirror. If the magnet is mounted in a horizontal plane so that it can be turned about a vertical axis, the perpendicular component of the intensity of magnetization can be studied for various relative positions of the magnet and crystal.

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[*] Weiss, Jour. de Phys., ser. 4, Vol. 4, p. 469, 1905.

Weiss' results show that this is a periodic function which repeats itself every 180°. The curve shown in Fig. 14 represents the ideal curve obtained in this manner. In practice the curve is not smooth but has irregularities.

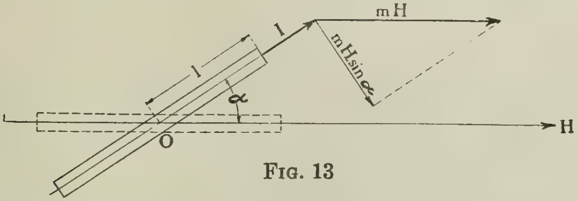

FIG. 13

15. *Component of Magnetization Parallel to the Field.*— If the disk is now placed between the poles of the magnet in a vertical position (see dotted line of Fig. 12) so that the plane of the disk makes an angle of 3° or 4° with the direction of the field H, and so that it can turn about a horizontal axis, it will be in a position to study the component of the intensity of magnetization which is parallel to the field.

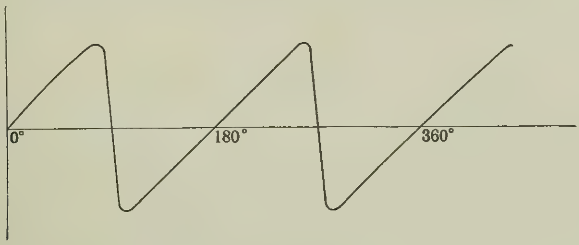

FIG. 14

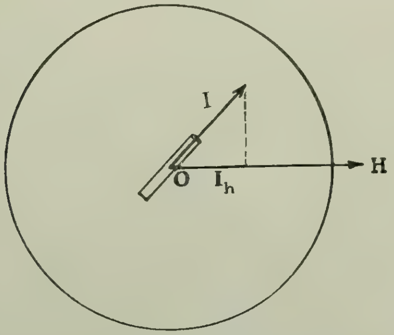

FIG. 15

Resolve the intensity of magnetization into a horizontal and a vertical component (see Fig. 15). Let I_h be the horizontal component. If the disk is placed so that I_h makes an angle of 3° or 4° with the direction of H, Fig. 16, there will be a restoring force tending to turn the disk back into a plane parallel to the field H. This can be measured in a manner similar to that shown in Fig. 12. The restoring force is

$$HI_h \sin \gamma = C\gamma'$$

where γ' is the deflection of the mirror. Therefore, if γ is kept constant the restoring force is proportional to I_h. Now $I_h \cos \gamma$, the component of I_h parallel to the field, is very nearly equal to I_h, since γ is only 3° or 4°. Hence, if we turn the disk through 180°, we obtain the components of the intensity of magnetization I_p, parallel to the field.

The component of the intensity of magnetization parallel to the field may also be studied by the method of induction by use of a solenoidal coil S, with a secondary S' inside. If the pyrrhotite disk d is thrust into the secondary with its plane parallel to the axis of the cylinder as shown in Fig. 17, there is a change of the flux through the secondary and hence a deflection of the ballistic galvanometer. The galvanometer will not be affected by the component of magnetization perpendicular to H, but only by that parallel to H. Therefore, by rotating the disk, one can study the relation between I_p and the angle of rotation of the disk. Weiss, by direct experiment, obtained results which are shown in Fig. 18, in which the abscissas are the azimuths of a constant magnetic field. The ordinates of the upper curve are the components of the magnetization parallel to the field, whereas those of the lower curve are the components perpendicular to the field. The phenomenon repeats itself every 180°.

In order to interpret these curves, Weiss makes use of the following illustration. Suppose a magnetic field is made to turn in the plane of an elliptic plate of soft iron. At the axes the magnetization coincides with the direction of the field. The longer axis will have a maximum, the shorter axis a minimum of magnetization. For all other directions of the field, the magnetization will be more nearly that of the longer axis than of the field. With the continuous rotation of the magnetic field, the magnetization will turn more slowly than the field in the neighborhood of the long axis and more rapidly in the neighborhood of the short axis. If the ellipse is very elongated, the component of magnetization perpendicular to the field will pass almost instantly from a very large negative value to a very large positive value, as represented in Fig. 14, while the direction of the field changes from one side to the other of the short axis. The lower curve of Fig. 18 may evidently be obtained by the addition of three curves, similar to that of Fig. 14, displaced with respect to each other by 60° and 120°.

From the above analogy, Weiss assumes that the complex structure of the crystal of pyrrhotite results from the juxtaposition of elementary crystals of which the magnetic planes are parallel and which possess each a direction of maximum and minimum magnetization at right angles to each other, and that these crystals are associated in the magnetic plane by the angles of 60°, or what amounts to the same thing, 120°.

The amplitudes of the abrupt variations of the component of magnetization perpendicular to the field give the relative importance of the three components of magnetization due to superposition of the crystals. The angles of the upper curve of Fig. 18 correspond to the minima of the magnetization parallel to the field of each of the components.

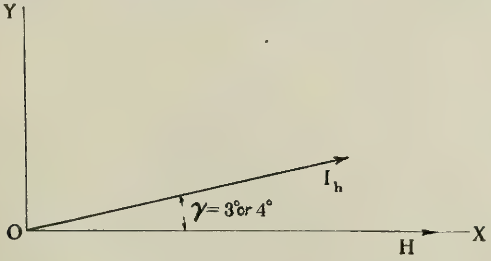

FIG. 16

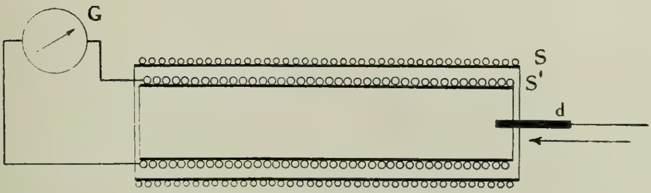

FIG. 17

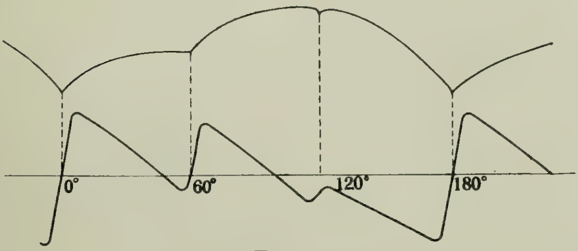

FIG. 18

16. *The Molecular Magnets of Pyrrhotite.*— In order to make a clear representation of the properties of a crystal of pyrrhotite, imagine that it be composed of rows of small needle magnets, equidistant from each other, pointing in the direction of easy magnetization Ox (Fig. 19), the axis Oy being the direction of difficult magnetization. Let the axis of rotation be perpendicular to the magnetic plane. Suppose that these magnets are small in comparison with the distances which separate them and strong enough to exercise on each other a directing action. Imagine moreover, that because of a compensation or of a greater distance, the rows do not affect one another. Left to themselves, the magnets of a row will adopt a position of equilibrium in which the north pole of each magnet will face the south pole of the following magnet. On this assumption, the substance would be saturated in the direction of easy magnetization Ox with little or no external field. Let a field H, making an angle α with the direction of the rows, act on the above system. The magnets will be deviated by an angle ϕ. As soon as the magnets are deviated from the direction Ox, there will be a force tending to restore

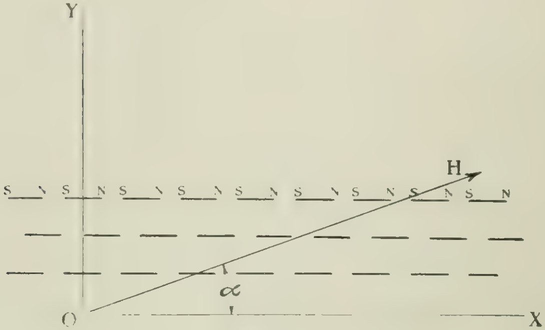

FIG. 19

them to their original position. If there were only two magnets, this force would take the direction indicated by the dotted line (1) Fig. 20, but the combined effect of all the magnets will give the force a direction (2), which is more nearly that of Ox. Let us assume that this force makes an angle ϕ with the direction Ox. The resultant action of the rows then exercises on each small magnet a magnetizing field $A\mu \cos\phi$ in the direction Ox, and in the direction Oy, a demagnetizing field $-B\mu \sin\phi$, where A and B are constants and where $\mu = ml$ is the magnetic moment of an elementary magnet. Let H_x and H_y be the sum of the components of the forces acting in the x and y directions respectively.

$$H_x = H \cos \alpha + A\,\mu \cos \phi \qquad (30)$$
$$H_y = H \sin \alpha - B\,\mu \sin \phi \qquad (31)$$

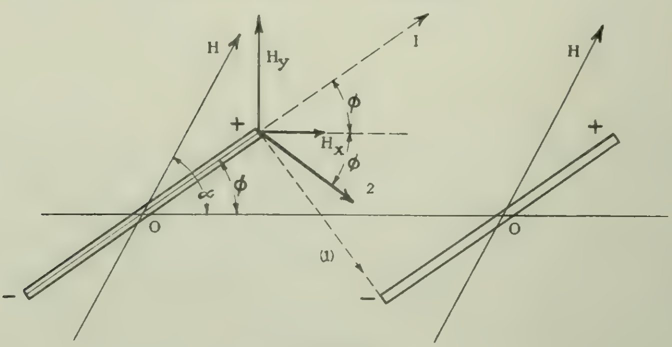

FIG. 20

For the condition of equilibrium

$$H_x \sin \phi = H_y \cos \phi$$

Therefore from (30) and (31)

$$(H \cos \alpha + A\mu \cos\phi) \sin\phi = (H \sin \alpha - B\mu \cos\phi) \cos\phi \qquad (32)$$

or,

$$H \sin (\alpha - \phi) = (A + B)\,\mu \sin\phi \cos\phi \qquad (33)$$

Writing

$$(A + B)\,\mu = NI_m$$

where N is a constant and I_m the intensity of magnetization per unit volume,

$$H \sin(\alpha - \phi) = NI_m \sin\phi \cos\phi \quad \text{.............} (34)$$

Thus with the above assumptions we arrive at an expression which as will be shown later, is the expression for the law of magnetization of pyrrhotite determined experimentally.

17. *Weiss' Law of Magnetization of Pyrrhotite.* — (a) Effect of Alternating Field.— It has been shown that the elementary magnets of pyrrhotite lie in the plane of the base of the crystal. The direction x is called the direction of easy magnetization; the perpendicular direction y, that of the difficult magnetization. If H is in the direction of easy magnetization, saturation takes place from the beginning. If, in this case, the field is reversed in direction the hysteresis curve will run through a cycle which is rectangular in shape (see Fig. 21). When the field has acquired a certain value $\pm H_c$, the elementary magnets become unstable and suddenly all of them swing around into the other position of equilibrium. Experiment gives very nearly a rectangle.

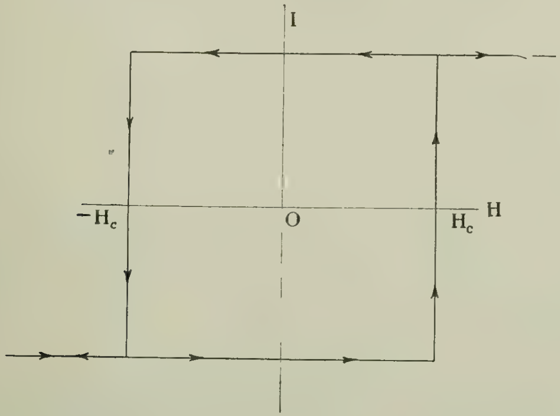

FIG. 21

(b) Effect of Rotating Field.— If a constant field is applied and rotated in the magnetic plane interesting phenomena present themselves. The elementary magnets have a tendency to remain in their original position so that when the field is rotated the direction of the intensity of magnetization turns slower than the field. If a field of 5,000 units acts in the direction Ox Fig. 22, and then rotates toward the direction Oy, the direction of the intensity of magnetization turns round much more slowly, and when H is in the direction Oy, I the intensity of magnetization, will be in the direction OI. Then when the field rotates a little farther, the magnets swing around very quickly so that, in the neighborhood of Oy, the rate of change of the direction of the intensity of magnetization is much greater than that of the field. Whatever the intensity of the field, the vector representing the intensity of magnetization falls within a circle C whose radius is equal to I_m, the maximum intensity of magnetization, which is the value obtained when the direction of the field coincides with the direction of easy magnetization. This circle is called the circle of saturation.

When the field is relatively weak, say 1,000 units, the intensity of magnetization vector follows the circle of saturation for only a short distance after which it describes a chord parallel to Ox. When the field has a strength of 7,300 units, or more, we have saturation in every direction of the field and the intensity of magnetization vector follows more closely the circle of saturation C.

Thus in order to turn and maintain the magnets in the direction of difficult magnetization Oy, it is necessary to apply in this direction a field of at least 7,300 units.

While for saturation in the direction Ox little or no field is required, the field necessary to produce saturation in the direction Oy must have a rather high value. Furthermore the intensity of magnetization remains constantly in the plane Oxy, the component of magnetization in the direction Oz perpendicular to this plane being very small in comparison with the magnetization in the characteristic plane. Thus while the demagnetizing field in the direction Oy is 7,300 units, that in the direction Oz is 150,000 units, or about 20 times greater.

Let α and ϕ be the angles which H and I make with Ox (Fig. 23): resolve H into components H_d directed along Oy and H_I parallel to the intensity of magnetization. There is a constant ratio

$$\frac{H_d}{I \sin \phi} = N \quad \text{.......................} (35)$$

between the components H_d of the field and the components of magnetization in the direction Oy. This fact was established experimentally by Weiss, a straight line relation between H_d and $I \sin \phi$ being obtained. The value of the constant N, for the sample that he used, was found to be

$$\frac{H_d}{I \sin \phi} = \frac{7300}{47} = 155.3$$

Everything takes place as if there were acting in the direction Oy, a demagnetizing force due to the structure of the crystal, proportional to the component of the intensity of magnetization in the direction of difficult magnetization Oy, and as if the remaining component of H were parallel to the direction of easy magnetization Ox.

From Fig. 23 we see that

$$H_d = AH = \frac{BH}{\cos \phi} = \frac{H \sin (\alpha - \phi)}{\cos \phi} \quad \text{............} (36)$$

Substituting (36) in (35) gives,

$$\frac{\dfrac{H \sin (\alpha - \phi)}{\cos \phi}}{I \sin \phi} = N$$

or

$$H \sin (\alpha - \phi) = NI \sin \phi \cos \phi \quad \text{..............} (37)$$

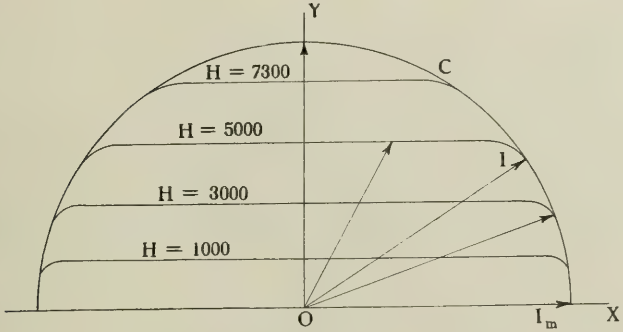

FIG. 22

This is the equation stating the law of magnetization of pyrrhotite, and is the same as that obtained when considering an elliptic plate of soft iron in a rotating field (Equation 34).

If, starting from the direction Ox, a field of 7,300 units or less has been rotated to the position Oy, the expression for the law of magnetization

$$H \sin(\alpha-\phi)=NI \sin\phi \cos\phi$$

becomes

$$H \cos\phi = NI \sin\phi \cos\phi$$

since in this position $\alpha = 90°$.

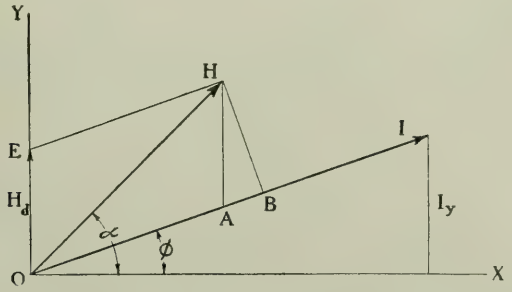

FIG. 23

For saturation this equation becomes

$$H \cos\phi = NI_m \sin\phi \cos\phi$$

therefore

$$\sin\phi = \frac{H}{NI_m}$$

For the same field there is a second position of equilibrium ϕ', Fig. 24, symmetrical to the first with respect to Oy. One might imagine that a coercive force H_c directed along $-Ox$ causes the various rows of the elementary magnets to turn from the first into the second position of equilibrium. This coercive force, in the case of pyrrhotite, is about 15 units whereas the demagnetizing field, or the field necessary to make the elementary magnets stand at right angles to the direction of easy magnetization, is about 7,300 units. The relation between the two quantities has not yet been explained, but it has been suggested that it has to do with the disturbed region of the extremity of the rows.

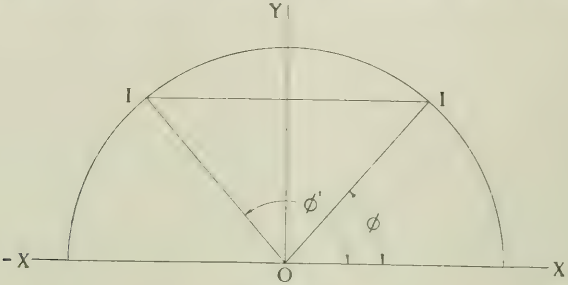

FIG. 24

18. *Magnetization in Different Directions.*— Let us now consider the laws of magnetization in directions other than that of Ox and Oy. As stated above, when a magnetizing field is applied in the direction of easy magnetization, saturation takes place from the beginning. If we plot intensity of magnetization I against field strength H, we will obtain the straight line AB (Fig. 25), parallel to the axis OH; that is, the intensity of magnetization remains the same whatever the value of the magnetizing field. Saturation occurs even without any external field.

Now if we apply a magnetizing force in the direction Oy, we obtain, from the general law of magnetization of pyrrhotite

$$H \cos \phi = NI \sin \phi \cos \phi$$

or

$$\begin{aligned} H &= NI \sin \phi \\ &= NI_y \end{aligned}$$

whence

$$\frac{H}{I_y} = N = \text{constant}$$

This holds for fields below that necessary to produce saturation and is represented by the line OC Fig. 25. For fields equal to or greater than that necessary to produce saturation the curve is the same as for easy magnetization. Therefore the whole curve for the intensity of magnetization when the field is applied in the direction Oy is given by OCB Fig. 25.

We have now to inquire what happens when the direction of the magnetizing field is intermediate between the direction Ox and Oy. We get different expressions for the law of magnetization depending on the way in which I is resolved. For a field of constant direction we may

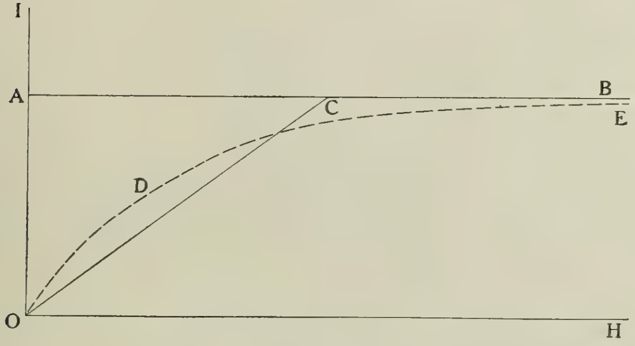

FIG. 25

take the component of the intensity of magnetization in the direction of the field. In this case (from Fig. 26)

$$I_h = I \cos(\alpha - \phi) \quad \ldots\ldots (38)$$

In general

$$H \sin(\alpha - \phi) = NI \sin\phi \cos\phi$$

For saturation this equation becomes

$$H \sin(\alpha - \phi) = NI_m \sin\phi \cos\phi \quad \ldots\ldots (39)$$

and (38) becomes

$$I_h = I_m \cos(\alpha - \phi) \quad \ldots\ldots (40)$$

Eliminating ϕ between (39) and (40) we find

$$H = \frac{N}{2} \sin 2\alpha \frac{I_h}{\sqrt{I^2_m - I^2_h}} - NI_h \cos 2\alpha - \frac{N}{2} \sin 2\alpha \sqrt{I^2_m - I^2_h}$$

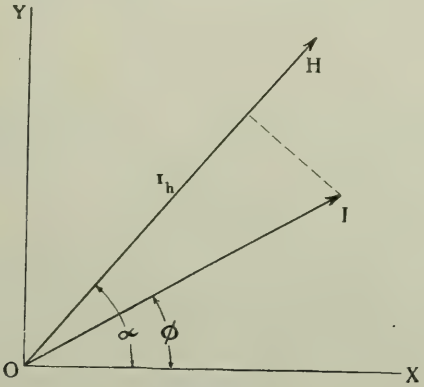

FIG. 26

If I_h approaches I_m, the first term of the right side of the equation approaches ∞; in other words, if H becomes ∞, I_h approaches I_m. Therefore the curve between the intensity of magnetization I_h, and the magnetizing force H approaches the line of saturation AB Fig. 25 asymptotically, and the law of magnetization for directions intermediate between Ox and Oy is given by the curve ODE Fig. 25. It is possible to explain all the ordinary curves of magnetization by a superposition of properties analogous to those of pyrrhotite in the direction of easy, difficult, and intermediate magnetization.

As has already been shown, the relation

$$H \sin(\alpha - \phi) = NI \sin\phi \cos\phi$$

deduced from the triangle OHE (Fig. 23), is the analytic expression of the law of magnetization. This expression multiplied by I gives the couple, or mechanical moment, exerted by the field on the substance.

$$\begin{aligned} M = HI \sin(\alpha - \phi) &= NI^2 \sin\phi \cos\phi \\ &= \frac{2NI^2}{2} \sin\phi \cos\phi \\ &= \frac{NI^2}{2} \sin 2\phi \quad \ldots\ldots\ldots\ldots\ldots (41) \end{aligned}$$

This couple is a maximum when

$$I = I_m$$
$$\phi = 45°$$

If we measure M as a function of α, the angle of orientation of the field with respect to the substance, we shall find a couple which becomes constant as soon as the field is strong enough to bring about saturation of the intensity of magnetization after this vector I has described an arc of 45° or more along the circle of saturation. This has been very clearly demonstrated by Weiss* in which he obtains the following curves (Fig. 27) experimentally. In these curves the angles of rotation of the field with respect to the substance are used as abscissas, and the couples, expressed in mm. divisions of the scale are used as ordinates. It will be noticed that the maxima for curves III, IV, and V are approximately the same, notwithstanding the fact that the magnetizing fields in the three cases are widely different. In order to reduce these couples to their absolute values per unit of volume it is necessary to multiply by 950.

From equation (41), the value of the maximum couple is $\frac{NI_m^2}{2}$. Taking $I_m = 47$, one obtains from the mean of the three values of this couple a demagnetizing field,

$$H_d = NI_m = 7300$$

which agrees with experimental determination.

19. *Hysteresis Phenomena.*—(a) Alternating Hysteresis.—We have seen that when a field $H = H_c = 15$ gauss is applied parallel to the direction of easy magnetization of pyrrhotite the elementary magnets turn completely over, yet it takes 7,300 gauss to make them stand at right angles to this direction.

Weiss has shown that, if one considers the direction of easy magnetization in a substance that is infinite and without fractures, the intensity of magnetization would remain constant as the magnetizing field increases and, if the magnetizing force were to describe a cycle, the theoretical hysteresis loop would be a rectangle as indicated by the

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* Weiss, Jour. de Phys., ser. 4, Vol. 4, p. 469, 1905.

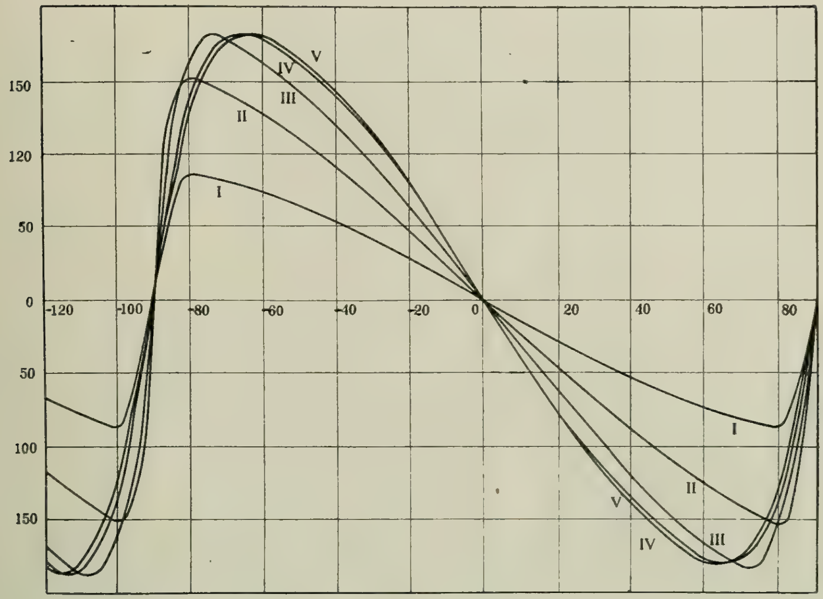

Fig. 27

Curve I. H = 1992 gauss
Curve II. H = 4000 gauss
Curve III. H = 7310 gauss
Curve IV. H = 10275 gauss
Curve V. H = 11140 gauss

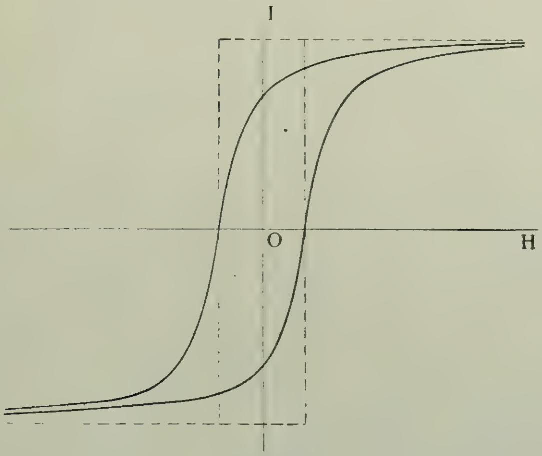

Fig. 28

dotted line in Fig. 28. The more uniform the magnetic material is, the less the experimental curve deviates from the theoretical curve. Weiss found that the distance between the ascending and descending branches of the experimental curve, measured parallel to the axis of abscissas, is approximately constant and equal to 30.8 gauss. Thus in order to move the vector of magnetization along the diameter of easy magnetization it is necessary to overcome a constant coercive field

$$H_c = 15.4 \text{ gauss.}$$

The energy dissipated per cycle in the form of heat in a unit of volume is

$$E_a = 4H_cI_m = 4 \times 15.4 \times 47$$
$$= 2900 \text{ ergs.}$$

(b) Rotating Hysteresis.— Excluding the direction of easy magnetization, the knowledge of magnetization in the magnetic plane has been obtained by causing a field of constant magnitude to turn in this plane. From these experiments one can obtain information on the form of hysteresis that has been called rotating hysteresis as distinguished from alternating hysteresis which has been considered above. If we rotate the field from the direction of easy magnetization OX (Fig. 29) through the angle XOY, the vector of the intensity of magnetization describes the arc AB of the circle of saturation. Then, the field passing the direction of difficult magnetization OY, the vector of the intensity of magnetization describes quickly the chord BFD, after which it describes the arc DE as the field describes the angle YO $(-X)$.

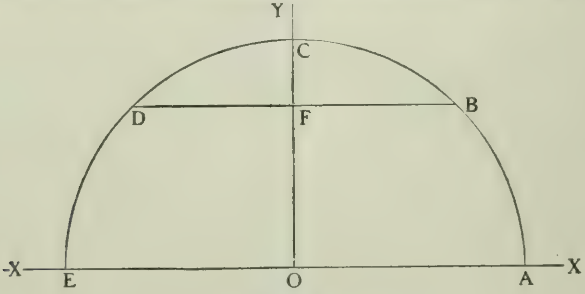

FIG. 29

Hysteresis occurs only in the direction of difficult magnetization OY when the molecules swing from one position of equilibrium into the other. In other words hysteresis accompanies the change of intensity of magnetization as the vector moves along BFD. When the magnetizing field H reaches the value 7,300 gauss the intensity of magnetization I follows the circle of saturation and the rotating hysteresis disappears. It follows from this that the hysteresis area along the circle of saturation is equal to zero. In the curves of Fig. 30, taken from experimental results obtained by Weiss with a sample of pyrrhotite in a field of about 600 gauss, 0 represents the region in the direction of easy magnetization and 90° that in the direction of difficult magnetization. It is seen that the two curves coincide exactly for some distance in the neighborhood of easy magnetization. In the neighborhood of the direction OY, marked by a, the two curves are distinctly different. The smaller the field with which one works the greater the divergence, but whatever be the field the curve corresponding to rotation in one direction can be superimposed on the return curve by a horizontal displacement.

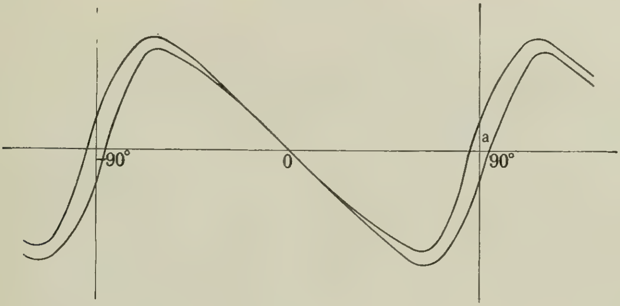

Fig. 30

20. *Energy of Rotating Hysteresis.*—So long as the intensity of magnetization I follows the curve of saturation there is no hysteresis loss. The loss takes place along SBS' Fig. 31. The force necessary to turn the elementary magnet over at S is less than at A. Call this force H'_c. Experiment has shown that

$$\frac{H_c}{I_m} = \frac{H'_c}{I_m - I_y} = \text{Const.}$$

therefore

$$H'_c = C(I_m - I_y).$$

To obtain the value of C let

$$I_y = O$$

then

$$H'_c = CI_m = H_c$$

or

$$C = \frac{H_c}{I_m}.$$

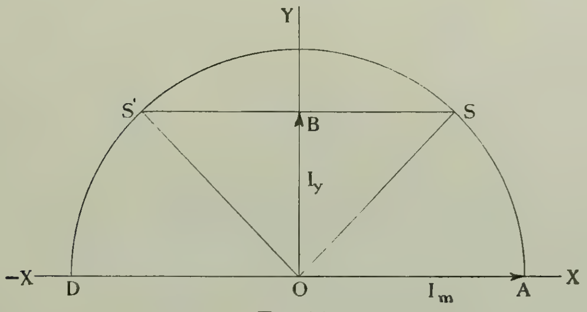

Fig. 31

Therefore the coercive force

$$H'_c = \frac{H_c}{I_m}(I_m - I_y).$$

If

$$I_y = I_m$$
$$H'_c = 0$$

and there is no hysteresis loss.

Now the energy dissipated in an alternating hysteresis, that is a hysteresis along AD (Fig. 31), was found above to be equal to $4\, I_m H_o$. The rotating hysteresis loss along SS' is $4\ H'_c BS$.

Now

$$(BS)^2 = I^2_m - I^2_y = I^2_m \left(1 - \frac{I^2_y}{I^2_m}\right)$$

whence

$$BS = I_m \sqrt{1 - \left(\frac{I_y}{I_m}\right)^2}$$

Therefore the energy dissipated in one rotating hysteresis cycle is

$$\begin{aligned} E_r &= 4H'_c BS \\ &= 4H'_c I_m \sqrt{1 - \left(\frac{I_y}{I_m}\right)^2} \\ &= 4\frac{H_c}{I_m}\left(I_m - I_y\right) I_m \sqrt{1 - \left(\frac{I_y}{I_m}\right)^2} \\ &= 4H_c I_m \left(1 - \frac{I_y}{I_m}\right) \sqrt{1 - \left(\frac{I_y}{I_m}\right)^2} \quad \text{..............} (42) \end{aligned}$$

If

$$I_y = 0$$

this reduces to the value for alternating hysteresis. Since the demagnetizing force, if the substance is infinite and continuous, is

$$H_d = NI_y$$
$$I_y = \frac{H_d}{N}$$

Substituting this in equation (42) we have

$$E_r = 4\, H_c I_m \left(1 - \frac{H_d}{NI_m}\right) \sqrt{1 - \left(\frac{H_d}{NI_m}\right)^2}$$

The energy E_a, dissipated in alternating hysteresis remains constant, whereas that of rotating hysteresis decreases with increase of I_y. This is shown graphically in Fig. 32 by the full line curve AB. The points (marked +) in the neighborhood of this curve are the results of measurements made by Weiss with a hysteresis-meter. They are subject to the correction $-nI_y$ since in a finite sample as was used in the experiment

$$H_d = (N + n) I_y$$

instead of

$$H_d = N I_y$$

where

$$n = \frac{H_I}{I}$$

in which H_I is the component of magnetization parallel to I.

21. *Magnetic Properties of Hematite.*— The magnetic properties of the crystal hematite are similar in many ways to those of the crystal pyrrhotite except that they are less pronounced. As in pyrrhotite, the elementary magnets of hematite lie in the plane of the base and it is paramagnetic perpendicular to this plane.

The crystal hematite has been studied by J. Kunz* and others. Kunz studied two distinct groups of crystals. The first group consisted of crystalline disks parallel to the base of the crystal. These crystals show a regular magnetic behavior and possess very small values of alternating and rotating hysteresis. The second group consisted of crystals composed of elementary magnets inclined toward each other at an angle of 60°. They possess so large an hysteresis effect that in many instances this overshadows the intensity of magnetization.

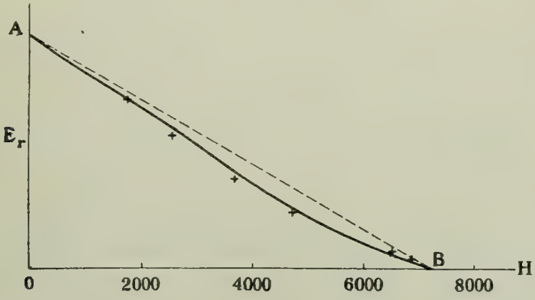

FIG. 32

It was found that the first group was not affected by temperature’ that is while the intensity of magnetization decreased with increase of temperature, becoming zero at 645°, yet when the temperature was again decreased the intensity of magnetization returned to its original values. This is not the case with crystals of the second group, for after the crystal has been heated to 650° its intensity of magnetization does not return to its original value on its return to lower temperatures.

The maximum value of the intensity of magnetization in the direction of easy magnetization is reached much more quickly in the case of pyrrhotite than in the case of hematite. In the case of alternating and rotating hysteresis, the effect is very much the same in the two crystals.

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*J. Kunz, Archives des Sciences, Vol. XXIII, 1907.

22. *Magnetic Properties of Magnetite.*— Besides the ferromagnetic crystals pyrrhotite and hematite there is a third, magnetite, whose magnetic properties are similar in many ways to those of the other two. It is classified as belonging to the regular system of crystals, but its magnetic properties indicate that it does not belong to this system. The magnetic properties of magnetite have been studied by Curie,* Weiss,† Quittner‡ and others, who found that they were more pronounced than those of hematite but less pronounced than those of pyrrhotite. From about 535° C. the temperature of magnetite transformation to 1375° C. the temperature of fusion of magnetite Curie found that the intensity of magnetization is independent of the field and that it decreases very regularly with the increase of temperature. For a part of this temperature range Curie formulated the following law: "The coefficient of magnetization of magnetite varies inversely as the absolute temperature between 850° C. and 1360° C."

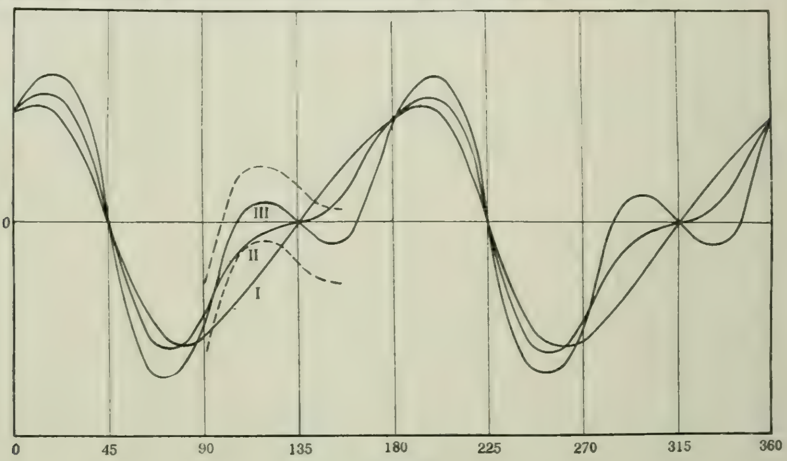

FIG. 33

Weiss and Quittner found that the magnetic properties indicate that the crystals of magnetite do not belong to the regular system. This was shown by taking a plate of magnetite cut parallel to the surface of the cube and placing it in the magnetic field in a horizontal position and then rotating the magnetic field round about it. Plotting their results with the angles of rotation of the magnetic field as abscissas, and as ordinates the deflection of the suspension which is proportional to the intensity of magnetization in a direction perpendicular to the magnetic field, they obtained the curves of Fig. 33. Each of these curves is the mean of the two curves obtained by rotating the field from 0° to 360° and then back to 0°. A portion of the two curves from which curve III is obtained is represented by the dotted lines. The area between the two curves is the hysteresis area of rotation.

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* P. Curie, Ann. de Chem., ser. 7, Vol. 5, p. 391, 1895.

† P. Weiss, Archives des Sciences, Vol. XXXI, 1911.

‡ V. Quittner, Archives des Science, Vol. XXVI, 1908.

If the symmetry were that of the cubic system the curve should show a period of 90°, thus forming between 0° and 360° four similar waves. Instead of this the figure shows only two identical waves between 0° and 360° for H = 57.3 gauss. The symmetry is therefore not that of the cubic system. Now the geometrical properties of the crystal leucite are such that it is usually classed as an isomeric crystal, yet when it is investigated optically it is found to be isotropic. However, as the temperature is raised, the optical properties also become those of the regular system. It would be interesting to see whether the properties of symmetry of magnetite, from the magnetic point of view, change with change of temperature. If according to its magnetic properties magnetite does not belong to the regular system, the question arises, to which system does it belong from a magnetic point of view?

The curves of Fig. 33 show that the two principal axes situated in the plane of the cubic plate are not equivalent. It remains to be determined whether the third axis, perpendicular to the plane of the plate, is equivalent to either of the other two or whether it behaves differently from the magnetic point of view. In the first case, we would have the symmetry of the quadric system, in the second, that of the othorombic system. Thus if the system is cubic, there should be between 0° and 180° three identical waves; if it is quadratic there should be two similar waves and a third which is different; and finally, for the symmetry of the othorombic system, the three waves should be different, with the possibility that one or even two of these waves may disappear completely. Quittner in his researches found the three waves to be different and that their relative magnitudes depend on the magnitude of the magnetic field. We must conclude therefore, that magnetite, so far as its magnetic properties are concerned, possesses the symmetry of the othorombic system.

In order to see the irregularities in the magnetic behavior of a crystal of magnetite and the dependence of the magnetic properties on the value of the magnetizing field, we need only to observe the curves, Fig. 34, obtained by Quittner with a plate cut from a crystal in such a way that its plane makes equal angles with the three axes. It will be noticed that for a field H = 167.2 gauss all three waves are practically equal; for a field H = 368.1 gauss there are only two waves which are greatly reduced and displaced, the third being barely visible. If the field is still increased one again finds, for H = 757 gauss, three well-defined waves which, however, are displaced by half a wave length.

The theoretical interpretation of these very complicated phenomena is hardly possible in the present state of our knowledge, but the investigators of magnetite have proposed that the crystal is made up of equal parts of three elementary magnets whose magnetic planes are perpendicular to each other, such as would be the case if it were possible to superimpose three plates of pyrrhotite cut parallel to the plane of easy magnetization in such a manner that their planes would be mutually perpendicular to each other.

III. EFFECT OF TEMPERATURE UPON THE MAGNETIC PROPERTIES OF BODIES

23. *Method of Investigation Used by Curie.*— Extensive investigations of the effect of temperature upon the magnetic properties of various substances have been made by Hopkinson,* Curie,† and others. Curie's method was to place the body to be tested in a non-uniform magnetic field and measure the resultant force of the magnetic actions by utilizing the torsion of a wire. Let *ABCD* (Fig. 35) represent the horizontal arms of an electromagnet, and let the axes of these two arms form an angle with each other. The body to be investigated is placed at the point *O* on the line *Ox*, which is the intersection of the horizontal plane passing through the axis of the arms of the electromagnet and the vertical plane of symmetry. When the electromagnet is excited, a force *f* of attraction or repulsion acts along *Ox*. Call H_y the intensity of the magnetic field at *O*. This field is directed, by reason of symmetry, along *Oy* perpendicular to *Ox*. Let *I* be the specific intensity of magnetization, that is the intensity of magnetization per unit mass, and *m* the mass of the body, then

$$f = mI\,\frac{dH_y}{dx}$$

If diamagnetic or paramagnetic bodies are being studied the demagnetizing force arising from the magnetization of the body is insignificant, and if *K* is used to designate the coefficient of specific magnetization, there is obtained

$$I = KH_y$$

and

$$f = mKH_y\frac{dH_y}{dx}$$

Now *K* for most diamagnetic and paramagnetic bodies is practically constant and therefore *f* is proportional to $H_y\,\frac{dH_y}{dx}$. For greatest sensitiveness the body should be placed at the point on *Ox* at which this product is maximum. Curie's method was to surround the sample under investigation by a vertical electric furnace so that it could be heated to any desired temperature. The sample itself was mounted on the end of a lever *lm*, which was suspended by a torsion wire *tm*. This lever was connected to another lever *mn* so that any movement of the sample would be greatly magnified at the other end of the system. The whole movable system was suspended in such a manner that any movement of the body, which was very small in the substances investigated by Curie, would be along *Ox*. With his apparatus, Curie claimed to be able to measure movements of the object to 0.001 mm. As the body was heated to various temperatures, which could be determined by means of a thermocouple, the forces of attraction or repulsion could be determined from the movement of the levers and the constant of the apparatus.

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* Hopkinson, Phil. Trans., p. 443, 1889.

† P. Curie, Ann. de Chem., ser. 7, Vol. 5, p. 289, 1895.

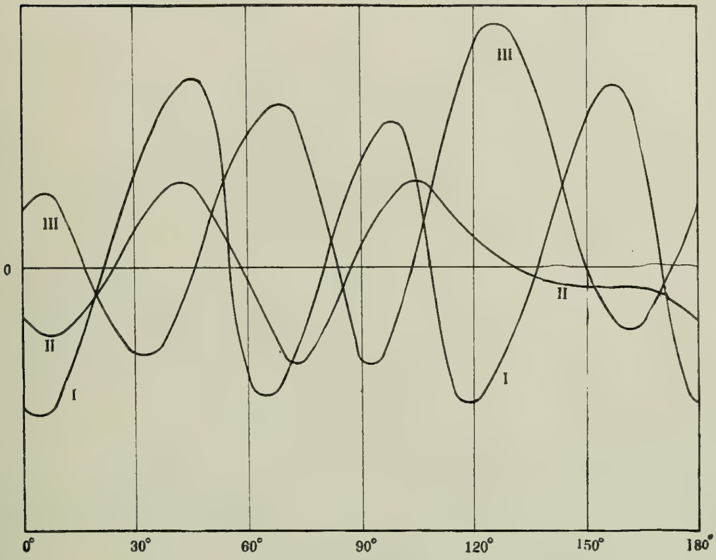

FIG. 34

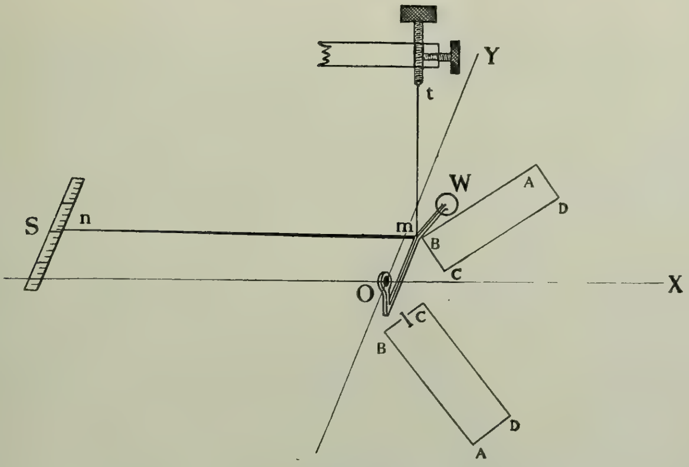

FIG. 35

24. *Results Obtained by Curie.* — After a very extended study, covering a wide range of substances — diamagnetic, paramagnetic, and ferromagnetic— Curie came to the following conclusions:

The coefficient of specific magnetization of diamagnetic bodies is independent of the intensity of the field, and as a general rule independent of the temperature. Antimony and bismuth are exceptions to this rule. The coefficient of magnetization of these bodies diminishes with increase of temperature. For bismuth the law of variation is a linear one. The physical and chemical changes of state often have only a slight effect on the diamagnetic properties. This is true in the case of the fusion of white phosphorus at 44°, and in the various transformations which are undergone when sulphur is heated. However, this is not always the case; the coefficient of magnetization of white phosphorus experiences a considerable diminution when this body is transformed into red phosphorus, antimony deposited by electrolysis in the allotropic state is much less diamagnetic than the ordinary variety, and the coefficient of magnetization of bismuth becomes, by fusion, twenty-five times more weak.

Paramagnetic bodies have also a coefficient of magnetization independent of the intensity of the field, but these bodies behave quite differently from the point of view of the changes produced by the change of temperature. The coefficient of specific magnetization varies simply in inverse ratio to the absolute temperature.

The difference in the effect of temperature on the coefficient of magnetization of magnetic and diamagnetic bodies is very marked, which is in favor of the theories which attribute magnetism and diamagnetism to causes of a different nature.

The properties of ferromagnetic and paramagnetic bodies are, on the contrary, intimately related. When a ferromagnetic body is heated it is transformed gradually and takes the properties of a paramagnetic body. The curve of Fig. 36 represents graphically the relation between temperature and intensity of magnetization I for a sample of iron subject to a magnetizing field of 1,300 units. In the region β, which commences at 760° C and extends to 920° C, the coefficient of specific magnetization obeys exactly an hyperbolic law up to 820°, after which it decreases more rapidly. Between 820° and 920°, at which point the γ state begins, it is probable that a gradual transformation takes place. In the γ state the iron possesses a susceptibility inversely proportional to the absolute temperature, which is characteristic of paramagnetic bodies. At 1280° where the last change of state takes place the coefficient of specific magnetization increases rapidly in the ratio of 2 to 3, after which it seems to take a variation in reverse ratio to the absolute temperature.

25. *Results of du Bois and Honda.*—H. du Bois and K. Honda* extended the investigations of Curie to a large number of elements

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* H. du Bois and K. Honda, Konink. Akad. Wetensch., Amsterdam, Proc. 12, pp. 596-602, 1909-1910.

(43 in all) and decided that Curie's conclusions do not admit of such extensive generalizations as have been given to them. They found that of the twenty or more diamagnetic elements examined there are only six which do not vary within the whole temperature range, and that during a change of physical state a discontinuity in the intensity of magnetization frequently occurs. This change may consist of a large or small break in the curve showing the relation between intensity of magnetization and temperature or of a rather sudden change in the shape of the curve.

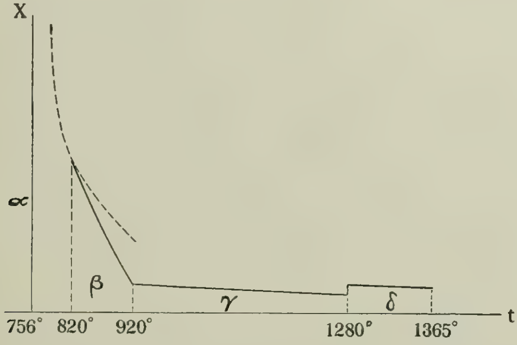

FIG. 36

Of the thermomagnetic examinations of polymorphous transformations made by du Bois and Honda, the most remarkable properties are shown by tin. They found that if diamagnetic grey tin is slowly heated, at 32° the intensity of magnetization, which is negative, suddenly changes (like the density) and at 35° passes through zero. Further heating continuously increases the magnetization until the value for paramagnetic tetragonal tin is reached at about 50°, after which it remains practically constant.

26. *Analogy Between the Manner in Which the Intensity of Magnetization of a Magnetic Body Increases under the Influence of Temperature and the Intensity of the Field, and the Manner in Which the Density of a Fluid Increases under the Influence of Temperature and Pressure.*— There are many analogies between the function $f(I, H, T) = O$ as applied to a magnetic body and the function $f(D, p, T) = O$ as applied to a fluid. The intensity of magnetization I corresponds to the density D, the intensity of the field H corresponds to the pressure p, and the absolute temperature T plays the same rôle in the two cases. For a paramagnetic body or a ferromagnetic body at a temperature above that of the transformation point, the relation is found

$$I = A\frac{H}{T}$$

where A is a constant. Similarly for a fluid sufficiently removed from its temperature of liquefaction one has the relation

$$D=\frac{I}{R}\times\frac{p}{T}$$

where $\frac{I}{R}$ is a constant. The law of the constancy of the intensity of magnetization when the field varies, and the inverse law of the absolute temperature for the coefficient of magnetization correspond respectively to the laws of Boyle and of Charles.

The manner in which the intensity of magnetization as a function of the temperature varies in the neighborhood of the temperature of transformation, the field remaining constant, corresponds to the manner in which the density of the fluid as a function of the temperature varies in the neighborhood of the critical temperature, the pressure remaining constant. The analogy between $\mathrm{I}=\phi$ (T) and $D=\phi$ (T) corresponding to pressures above the critical pressures is shown graphically in Figs. 37 and 38. Although as Curie has shown the analogy seems to be almost perfect when the field strength in the one case and pressure in the other is kept constant, yet it has not been shown to hold in the case where temperature is kept constant in both phenomena.

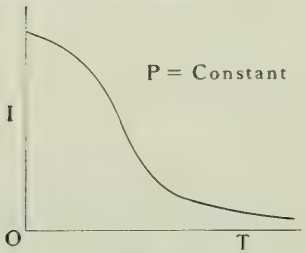

Fig. 37

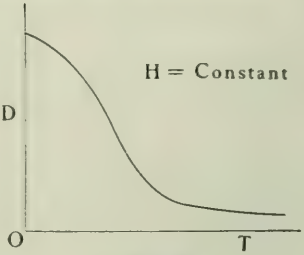

Fig. 38

In the case of magnetization if the temperature is kept constant and the field strength alternated a hysteresis loop is obtained when intensity of magnetization I is plotted against field strength H. That this is true in the case of a fluid when the temperature is kept constant and the relation between density and pressure plotted has not been shown. It is true that in the case of solids the phenomena of lag occurs with variation of density and pressure at constant temperature, and it may be true to a small degree in the case of liquids, but it is hard to conceive of it as being true in the case of gases.

IV. Experimental Evidence in Favor of the Electron Theory of Magnetism

27. *The Molecular Magnetic Field of Pyrrhotite.* — Let us assume that we have two fields, H and H_m, acting in a crystalline substance and that the crystalline structure possesses three rectangular planes of symmetry coincident with the planes of the system of coördinates, and that each component of the molecular field is proportional to the corresponding component of the intensity of magnetization with a coefficient

$N_1 > N_2 > N_3$ for each axis. Then the molecular field H_m has, in general, not the direction of the intensity of magnetization except when the latter is directed along one of the axes. Thus the intensity of magnetization assumes of itself the direction of one of the axes, Ox, Oy, or Oz, Fig. 39, and this orientation will correspond to a stable equilibrium only for the axis Ox for which the coefficient N_1 is the greatest.

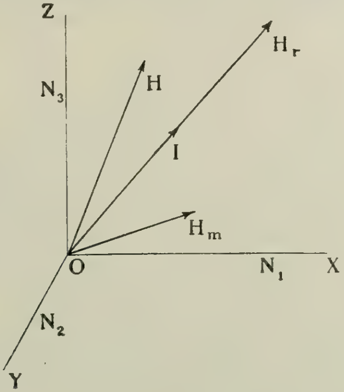

FIG. 39

If an external field H is added to the molecular field the magnetization assumes the direction of the resultant magnetic field. Now

$$H_x + H_{mx} = H_x + N_1 I_x,$$
$$H_y + H_{my} = H_y + N_2 I_y$$
$$H_z + H_{mz} = H_z + N_3 I_z$$

If the resultant H_r coincides with I in one direction,

$$\frac{H_x + N_1 I_x}{I_x} = \frac{H_y + N_2 I_y}{I_y} = \frac{H_z + N_3 I_z}{I_z}$$

and if the magnetization takes place in a plane, say the xy plane,

$$\frac{H_x + N_1 I_x}{I_x} = \frac{H_y + N_2 I_y}{I_y}$$

Let α and ϕ, Fig. 40, be the angles which H and I make with the axis Ox.

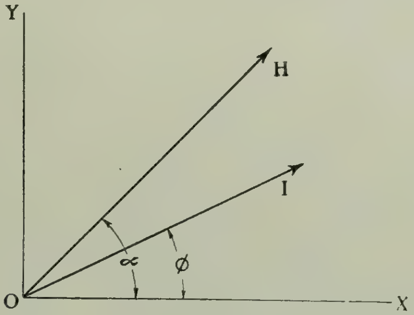

FIG. 40

Then

$$\frac{H \cos\alpha + N_1 I \cos\phi}{I \cos\phi} = \frac{H \sin\alpha + N_2 I \sin\phi}{I \sin\phi}$$

whence

$$IH \cos\alpha \sin\phi + N_1 I^2 \cos\phi \sin\phi = IH \sin\alpha \cos\phi + N_2 I^2 \sin\phi \cos\phi$$

Transposing and simplifying,

$$IH \sin(\alpha - \phi) - (N_1 - N_2) I^2 \sin\phi \cos\phi = 0$$

or

$$H \sin(\alpha - \phi) = (N_1 - N_2) I \sin\phi \cos\phi$$

Put

$$N_1 - N_2 = N$$

and

$$H \sin(\alpha - \phi) = NI \sin\phi \cos\phi$$

This equation expresses the law of magnetization of pyrrhotite, (see equation (37), page 9) as determined experimentally by Weiss. Therefore the above hypothesis accounts for the experimental properties of pyrrhotite in the *xy* plane. These properties are different from those in the *xz* plane only in the magnitude of the constant.

The equation

$$HI \sin(\alpha - \phi) = (N_1 - N_2) I^2 \sin\phi \cos\phi$$

states that the couple $HI \sin(\alpha - \phi)$ exerted by the external field on the intensity of magnetization is equal to the couple $(N_1 - N_2) I^2 \sin\phi \cos\phi$, which is due to the structure of the crystal and which would remain if the external field were suppressed. The latter couple tends to bring the elementary magnets back into the direction of easy magnetization. The position of equilibrium corresponding to the orientation of the magnets in the direction Ox is stable only when $N_1 > N_2 > N_3$.

28. *Variation of the Intensity of Magnetization of Magnetite with Temperature.*— It has been shown, page 21, that

$$\frac{I}{I_m} = \frac{\cosh a}{\sinh a} - \frac{1}{a} \quad \text{.....................} (28)$$

where

$$a = \frac{MH}{RT} \quad \text{.......................} (43)$$

In the case of ferromagnetic substances we have, in addition to the external field, an anterior or molecular field H_m which is due to the action of the molecules upon each other and which has been called by Weiss the "intrinsic molecular field." Just as liquids can exist when the external pressure is zero, so ferromagnetic bodies can take a finite intensity of magnetization in the absence of exterior fields. If the interior field acted alone the intensity of magnetization would be proportional to it and we would have

$$H_m = NI$$

and equation (43) would become

$$a = \frac{NMI}{RT}$$

or

$$I = \frac{aRT}{MN} \quad \text{.......................} (44)$$

where N is the factor of proportionality.

This equation is represented in Fig. 41 by the straight line OA while equation (41) is represented by the curved line OBA. The intensity of magnetization being satisfied by equations (41) and (44), the points of intersection of the curve and the straight line give the values of I. One solution of these equations is

$$I = 0$$

$$a = 0$$

from which it follows that

$$H = 0$$

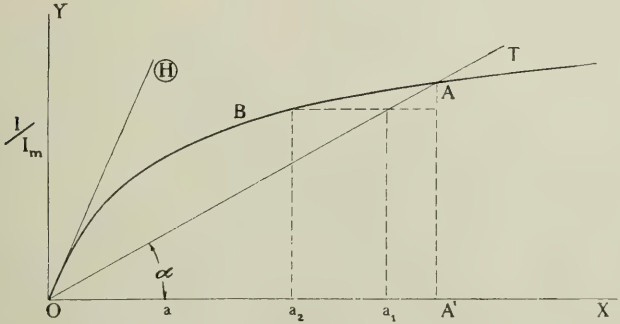

FIG. 41

It can be shown, however, that only the point A represents a state of stable equilibrium of magnetization. For suppose that we were able to decrease the intensity of magnetization I directly. For the same value of I and T, the value of a given by the straight line (namely a_1) is larger than a_2 given by the curve (See Fig. 41). a_1 is the value due to the molecular field which is much stronger than the external field whose action is represented by the curve and by a_2. Now a decrease in the intensity of magnetization means that fewer elementary magnets have their magnetic axes pointing in the same direction. Referring to Fig. 20, it will be seen that if the molecular field or the field due to the action of one elementary magnet on another, is stronger than the external field H, equilibrium will be established only when the magnets have oriented themselves into a position such that the components of the moments of the two fields are the same, that is, when the intensity of magnetization has risen to the point A. On the other hand the point O represents a value at which the intensity of magnetization is zero, that is, where as many elementary magnets point in the direction of easy magnetization as in the opposite direction. A slight mechanical or magnetic disturbance will cause half of the elementary magnets to swing around into the direction of easy magnetization where they remain in stable equilibrium. Thus the point O corresponds to unstable magnetization and the point A to stable magnetization.

As the paramagnetic susceptibility is very small enormous magnetic fields would be required to increase this spontaneous magnetization which is due to the external field. Assuming the mechanical analogy of Weiss in which the increase in the density of a liquid requires external pressures that are incomparably greater than those by means of which the density of gas is changed, we arrive at the conclusion that for the absolute temperature T, the ordinate $A'A$ represents the saturation value of the intensity of magnetization. According to this a ferromagnetic substance is saturated without the least external field. Though this inference, drawn from the theory, seems in contradiction to the larger number of experimental facts observed, yet it agrees perfectly with the phenomena of magnetization observed in crystals, and especially in those of normal pyrrhotite. This crystal has the very valuable property of a magnetic plane in which all the elementary magnets are situated. In this magnetic plane is a direction in which saturation takes place with very little or no external field, while a field of about 7,300 units is required to bring about saturation in a perpendicular direction. The assumption of a molecular field accounts very satisfactorily for the laws that govern the magnetization of the normal pyrrhotite as a function of the external field. The intensity of magnetization as a function of the temperature is a very complicated phenomenon, varying from one substance to another, and also varying in the same substance with the magnetic field; therefore it appears doubtful whether the most simple hypothesis of the uniform molecular magnetic field will be able to account for all the observed phenomena. In the case of magnetite, however, Weiss has shown that the theoretical curve coincides with the experimental curve between the temperatures — 79° C. and + 587° C. On the assumption that a piece of ordinary iron is composed of small crystals having the property of a magnetic plane, Weiss has also shown that the hysteresis loops of annealed iron can be given a theoretical interpretation.

In order to determine the absolute values of the internal magnetic field we have to examine the magnetic properties of the ferromagnetic substances in the neighborhood of the point where the spontaneous magnetization disappears. Thus iron loses its spontaneous magnetization at the temperature of 756° C. Between this point and 920° C. iron has still a considerable susceptibility, the magnetism, however, appearing only under the combined action of the external and internal fields. In this region we have

$$a=\frac{M(H_e+NI)}{RT} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

or

$$T=\frac{M(H_e+NI)}{aR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

where M is the resulting magnetic moment of each molecule and H_e the external field. Equation (29), page 21, gives

$$\frac{I}{I_m}=\frac{\cosh a}{\sinh a}-\frac{1}{a}=\frac{1}{3}a-\frac{2}{90}a^3+\frac{4}{45.42}a^5-\ldots$$

As long as we consider only the beginning of the curve OB of Fig. 41 whose tangent at the origin corresponds to the temperature $\Theta = 756 +$

273 = 1029, we may consider only the first term on the right-hand side of the last equation. Then

$$I/I_m = a/3 \tag{47}$$

Up to the temperature Θ we have spontaneous ferromagnetism where the external field H_e is negligible in comparison with NI the internal magnetic field, so that in equation (45), we may write

$$a = \frac{MNI_m a}{3R\Theta}$$

whence

$$\Theta = \frac{MNI_m}{3R} \tag{48}$$

Dividing (46) by (48)

$$\frac{T}{\Theta} = \frac{3H_e}{aNI_m} + \frac{3I}{aI_m}$$

or from (47)

$$\frac{T}{\Theta} = \frac{3H_e}{a\,NI_m} + 1$$

Since

$$I_m = \frac{3}{a} I$$

this equation reduces to

$$\frac{T - \Theta}{\Theta} = \frac{H_e}{NI}$$

or

$$(T - \Theta)\, I = \frac{\Theta H_e}{N} \tag{49}$$

This equation represents an equilateral hyperbola and allows us to determine the coefficient N. Weiss found for

iron	$N = 3{,}850$	$H_m = 6{,}560{,}000$
nickel	$N = 12{,}700$	$H_m = 6{,}350{,}000$
magnetite	$N = 33{,}200$	$H_m = 14{,}300{,}000$

If the molecular magnets act upon one another with magnetic forces of this enormous amount, the potential energy due to the molecular magnetic field must have very large values.

29. *Specific Heat and Molecular Field of Ferromagnetic Substances.* — The mutual energy of a number of magnets of invariable magnetic moment M is

$$W = -\tfrac{1}{2} \Sigma MH \cos \alpha$$

H being the field in which is placed one of the elementary magnets and which is due to all the other magnets and α being the angle between H and M. When this summation is extended to all the elementary magnets contained in 1 cc. H becomes the molecular field H_m and the intensity of magnetization I is the geometric sum of the magnetic moments M. The energy of magnetization per unit of volume is then

$$W = -\tfrac{1}{2} IH_m.$$

The molecular field is related to the intensity of magnetization I by

$$H_m = NI$$

where N is a constant coefficient. Therefore

$$W = -\tfrac{1}{2} NI^2.$$

Since this energy is negative it is necessary to add energy in order to demagnetize. Thus the intensity of magnetization decreases continuously as the temperature increases from absolute zero to the temperature Θ of the disappearance of spontaneous ferromagnetism. The total quantity of heat absorbed by the magnetic phenomena per unit of mass of the body, between the temperature where the intensity of magnetization is I and the temperature Θ is

$$q_m = \tfrac{1}{2}\frac{NI^2}{JD}$$

where J is the mechanical equivalent of the calorie and D is the density.

The specific heat due to change in the intensity of magnetization is then

$$s_m = \frac{dq_m}{dt} = \frac{1}{2}\,\frac{N}{JD}\,\frac{\partial I^2}{\partial t}$$

which must be added to the ordinary specific heat. According to Curie's experimental results I, the intensity of magnetization for iron at ordinary temperatures, is equal to 1700 and

$$N = 3850.$$

Therefore

$$H_m = NI = 6{,}540{,}000.$$

The energy of diamagnetization per unit mass is

$$W = \tfrac{1}{2} NI^2 = 70.6 \times 10^7 \text{ ergs.}$$

At 20° C.

$$q_m = \frac{70.6 \times 10^7}{4.19 \times 10^7} = 16.8 \text{ calories.}$$

The following results for iron were obtained from Curie's experimental data.

t°C.	I/D C. G. S.	q_m cal.	$s_m = \frac{\Delta q_m}{\Delta t}$	In the Interval
20	216.3	16.8		
275	207.5	15.5	.005	20° – 275°
477	189.6	12.9	.013	275° – 477°
601	164.0	9.7	.027	477° – 601°
688	127.	5.8	.045	601° – 688°
720	100.7	3.6	.068	688° – 720°
740	64.	1.5	.108	720° – 740°
744.6	50.1	.9	.136	740° – 744.6°
753	0. (by extrapolation)			

From this we see that at ordinary temperatures the specific heat is altered, due to the magnetic phenomena, by only a small amount, while in the neighborhood of Θ, the temperature at which spontaneous magnetization disappears, the effect amounts to 0.136, or about two fifths of the total value.

The results of Weiss and Beck * show a very interesting relation between the variations of the ordinary specific heat and

$$s_m = \frac{\Delta q_m}{\Delta t}$$

as functions of the temperature. Their results are represented graphically in Fig. 42, curve *A* representing the relation between ordinary specific heat and temperature and curve *B* that between s_m, the specific heat due to magnetization, and temperature. The same close agreement has been found in the ferromagnetic substances, nickel and magnetite.

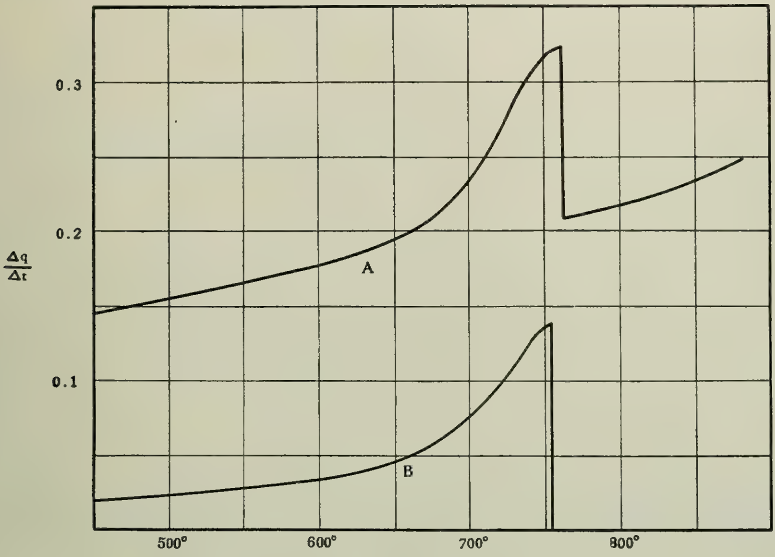

FIG. 42

30. *The Elementary Magnets of Iron, Nickel, and Magnetite.*—The electron theory of ferromagnetism gives us a means of determining the moment *M* of the molecular magnets of those substances whose internal magnetic field has been determined. Various methods may be applied for this purpose, but the one used by J. Kunz† is probably the most simple. In his method Kunz makes use of the equation

$$a = \frac{MNI}{RT} \quad \ldots\ldots (50)$$

where *N* is a constant. For the absolute temperature Θ, the temperature at which the spontaneous ferromagnetism disappears, we have the relation

$$I/I_m = a/3 \quad \ldots\ldots (51)$$

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* Weiss and Beck, Jour. de Phys., ser. 4, Vol. 7, p. 249, 1908.
† J. Kunz, Phys. Rev., Vol. XXX, No. 3, March, 1910.

Substituting (51) in (50), we find

$$a = \frac{MNI_m a}{3\ RT}$$

or

$$M = \frac{3\ R\Theta}{NI_m} \quad \text{.......................} (52)$$

where Θ is the particular temperature considered. R is given by the equation of gases

$$p = RN_1T$$

where

$$p = 10.1 \times 10^6$$
$$T = 273$$
$$N_1 = 2.7 \times 10^{19}$$

Therefore

$$R = 1.36 \times 10^{-16}.$$

I_m, the saturation value of the intensity of magnetization at the absolute zero of temperature, has to be determined from the above theory by means of the value I, the intensity of magnetization for the case of saturation at the temperature $t°$.

In the case of iron, we have from Curie's results, $I = 1700$ for ordinary temperatures and a field strength of 1300 units, and $I_m = 1950$. Taking $N = 3850$, the value found by Weiss and Beck,* and $\Theta = 756 + 273 = 1029$, we find, by substitution in equation (50) $M = 4.445 \times 10^{-20}$ absolute electromagnetic units.

Let N_1 be the number of molecular magnets in unit volume at the absolute zero. Then we have

$$N_1M = I_m = 1950$$

or

$$N_1 = 4.386 \times 10^{22}.$$

If this number N_1 of elementary magnets is at the same time the number of molecules of iron, and if the mass of one molecule of iron is equal to μ, we have

$$N_1\mu = \delta = 7.36$$

where δ is the density of iron at the absolute zero.

$$\mu = \delta / N_1 = 1.792 \times 10^{-22} \text{ grams}$$

Let us assume that the molecule of iron consists of two atoms, then it will be 111.8 times heavier than the atom of hydrogen, and the mass of the atom of hydrogen μ_H will be equal to

$$\mu_H = \frac{1.792 \times 10^{-22}}{111.8} = 1.603 \times 10^{-24} \text{ grams}$$

Du Bois and Taylor Jones† found the intensity of magnetization of iron continues to increase up to field strengths of 1500 units. At this value they found the value of I for ordinary temperatures to be 1850. Substituting this value instead of 1700, the value obtained by Curie, we have

$$\mu_H = 1.66 \times 10^{-24} \text{ grams}$$

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* Weiss and Beck, Jour. de Phys., Vol. 7, pp. 249, 1908.

† Du Bois and Taylor Jones, Elektrot. Zeitschr., Vol. 17, p. 544, 1896.

A recent value of this quantity has been determined by Rutherford by means of radioactive phenomena. He found

$$\mu_H = 1.61 \times 10^{-24} \text{ grams}$$

The close check of the values for the mass of one atom of hydrogen deduced from so very different phenomena as those of radioactivity and ferromagnetism by considerations entirely different, is strong evidence in favor of the theories involved. It follows from the above that the molecular magnet of iron contains two atoms.

In the case of magnetite, Fe_3O_4, we find the following numbers:

$$N = 33{,}200$$
$$I = 430$$
$$I_m = 490$$
$$\delta = 5.2$$

According to Curie

$$\Theta = 536 + 273 = 809.$$

Using these values we find

$$M = 2.02 \times 10^{-20}.$$

Now

$$N_1 M = I_m = 490$$

therefore

$$N_1 = 2.42 \times 10^{22}$$

If the mass of each molecule is equal to μ we have

$$N_1 \mu = 5.2$$

whence

$$\mu = 2.15 \times 10^{-22}.$$

Now

$$Fe_3 0_4 = 231.7$$

therefore

$$\mu_H = \frac{2.15 \times 10^{-22}}{231.7} = 0.93 \times 10^{-24} \text{ grams,}$$

a value that agrees pretty well with that obtained from other sources when we take into consideration the comparatively small amount of work that has been done on magnetite.

According to Weiss

$$\Theta = 589 + 273 = 862,$$

instead of 809 as given above. Substituting this value for Θ we find

$$\mu_H = 0.99 \times 10^{-24} \text{ grams.}$$

In the case of nickel we have the following values:

$$N = 12{,}700$$
$$I = 500$$
$$I_m = 570$$
$$\Theta = 376 + 273 = 649$$
$$M = 3.65 \times 10^{-20}$$
$$N_1 = 1.56 \times 10^{22}$$
$$N_1 \mu = \delta = 8.8$$
$$\mu = 5.63 \times 10^{-22}$$

The atomic weight of nickel is 5.87, hence, assuming that each molecular magnet contains two atoms, we find

$$\mu_H = \frac{5.63 \times 10^{-22}}{117.4} = 4.8 \times 10^{-24},$$

a value that is just three times larger than that given by Rutherford. As the degree of accuracy is the same in the case of iron, nickel, and magnetite, the experimental evidence indicates that the molecular magnet of nickel is made up of six atoms, or that the number of degrees of freedom is only one third as great as in the case of iron. In a recent investigation, Stifler* has determined the above quantities for cobalt. He obtained the following values:

$N = 6{,}180$
$I = 1{,}310$
$I_m = 1{,}435$
$\theta = 1{,}348$
$M = 6.21 \times 10^{-20}$
$N_1 = 2.31 \times 10^{22}$
$N_1\mu = \delta = 8.77$
$\mu = 3.8 \times 10^{-22}$

Since the atomic weight of cobalt is 59, we obtain, on the assumption that each molecular magnet contains two atoms,

$$\mu_H = \frac{3.8 \times 10^{-22}}{118} = 3.22 \times 10^{-24}$$

a value that is just two times larger than Rutherford's value. If we apply the above reasoning we must conclude that the elementary magnet of cobalt is made up of four atoms.

The quantities considered above are given in the following table: *n* is the number of atoms corresponding to one elementary magnet.

Substance	$I_t = 20°$	I_m	θ°C.	N	$NI = H_m$	$M \times 10^{20}$	$\mu_H \times 10^4$	n
Fe	1,700	1,950	756	3,850	6,540,000	4.445	1.603	2
Fe_3O_4	430	490	536	33,200	14,300,000	2.02	0.93	
Ni	500	570	376	12,700	6,350,000	3.65	4.8	6
Co	1,421	1,435	1,075	6,180	8,870,000	6.21	3.22	4

As the ratio of the density of nickel and iron, 1.12, is nearly equal to the ratio of the atomic weights, 1.05, of the two metals, the number of molecules per unit volume must be the same for both metals, assuming that each molecule contains two atoms. Since the moment of the molecular magnets of nickel is only about 18 per cent smaller than that of iron, we should expect that the intensity of magnetization of nickel would vary by about this amount from that of iron, while in reality the magnetization of iron is 3.4 times greater than that of nickel. This consideration indicates again, that either the molecular magnet of nickel contains six atoms or that only every third molecule is an elementary magnet.

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* W. W. Stifler, Phys. Rev., Vol. XXXIII, No. 4, p. 268, 1911.

This fundamental difference in the molecular magnets of iron and nickel must be taken into account when explaining some of the very interesting differences in the magnetic behavior of the two metals. Thus, the first layer of electrolytically deposited nickel is stronger magnetically than the subsequent layers, while for iron the opposite is true, that is, thin layers of iron are much less magnetic than thicker layers. In addition, a longitudinal compression decreases the magnetization of iron and increases that of nickel. In a recent article the author has shown * that the effect of transverse joints in nickel bars is to increase the magnetic induction rather than decrease it as in the case of iron.

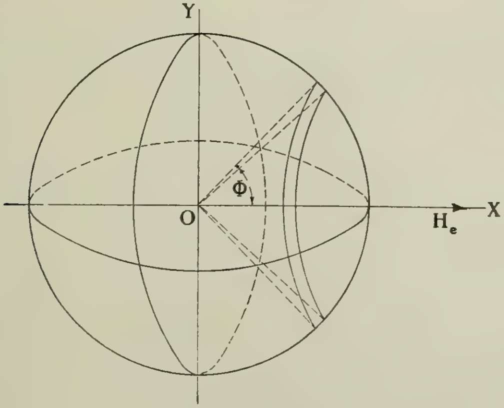

FIG. 43

31. *The Hysteresis Loop of Iron.*— Let us assume that the elementary crystal of iron has properties analogous to those of the crystal of pyrrhotite and that the direction of easy magnetization is distributed uniformly throughout the volume. Working with weak fields let us first consider only the irreversible phenomena. When the substance is in the neutral state, the magnetization vectors of the different elementary crystals will terminate on the surface of a sphere with uniform density. If the field H acting in the direction Ox, Fig. 43, exceeds the coercive field H_c, all the elementary magnets which were originally directed in the negative direction will swing round so that all the intensity of magnetization vectors will be contained in a cone having OH for its axis and of semiangle ϕ which is given by

$$\cos\phi = H_c/H.$$

Each of the elementary magnets that swings around will contribute its moment M_1 to the resulting intensity of magnetization in the direction x. Now the number of vectors ending on the sphere is equal to N, the number of elementary crystals with a plane of magnetization, and the number ending on the zone subtended by the angle ϕ before the field is applied will be

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* E. H. Williams, Phys. Rev., Vol. XXXIII, No. 1, p. 59, 1911.

$$\frac{2\pi r \sin\phi \, d\,\phi}{4\pi r} N = N/2 \sin\phi \, d\phi$$

The moment, due to these magnets, in the direction Ox is

$$M_x = M_1 \cos\phi \, \frac{N}{2} \sin\phi \, d\phi.$$

The moment due to all the magnets that swing round into the direction Ox is

$$\begin{aligned} M &= \int_o^\phi \frac{M_1 N}{2} \sin\phi \cos\phi \, d\phi. \\ &= \frac{M_1 N}{4} \sin^2\phi \\ &= \frac{I_m}{4} \sin^2\phi \end{aligned}$$

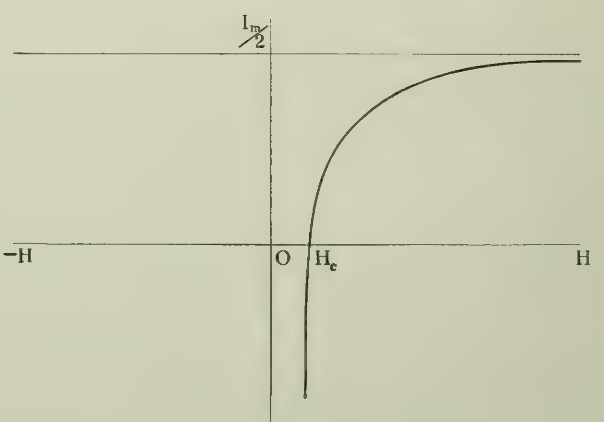

Fig. 44

since the number originally in the positive direction is equal to the number that have been turned around, the resulting moment in the direction Ox will be

$$\begin{aligned} M &= 2\,\frac{I_m}{4} \sin^2\phi \\ &= \frac{I_m}{2} \sin^2\phi \end{aligned}$$

Now

$$\cos\phi = H_c/H$$

therefore

$$M = \frac{I_m}{2}\,[1-(H_c/H)^2] \quad \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

where M is the resultant magnetic moment per unit volume, that is, the intensity of magnetization I.

The graphic representation of the equation

$$I=I_m/2\left[1-(H_c/H)^2\right]\ldots\ldots\ldots\ldots\ldots\ldots(54)$$

is given by Fig. 44. This resembles an hyperbola whose horizontal asymptote is given by $I=\frac{I_m}{2}$ and whose vertical asymptote is given by $H = 0$. If H is equal to H_c the intensity of magnetization $I = 0$ and if $H = \infty$, $I = I_m/2$. But physically this must be the value of saturation I_m therefore we shall write

$$I=I_m\left[1-(H_c/H)^2\right]\ldots\ldots\ldots\ldots\ldots\ldots(55)$$

If we were to draw the curve corresponding to this equation, we should find a curve of exactly the same character as the previous one except that for a given value of H the ordinate I would be twice as large as before.

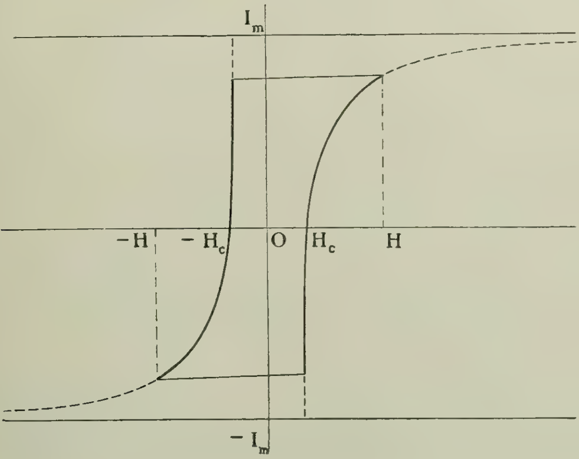

Fig. 45

If one causes the field to oscillate between the value $+H$ and $-$ H, the graphical representation of the curve of equation (55) will be given by Fig. 45. When the field $-H$ is applied, the figurative points will be collected on the negative side of the sphere of Fig. 43. With the sample in this condition let us begin the description of a cycle, varying the field from $-H$ to increasing positive values. The intensity of magnetization will change very little so long as the field is less than $+H_c$. At this point it begins to change very rapidly and will describe a curve similar to the curves considered above. This curve with the portion of the straight line already described will constitute half of a cycle corresponding to a variation of the field from $-H$ to $+$H. The cycle is completed, from symmetry, by returning to the origin.

Equation (55) assumes that the coefficients N_2 and N_3 of Fig. 39 are zero. Now, in the case of iron, this is only approximately true. Making these corrections, the theoretical considerations give cycles which are shown in Fig. 46. The scale has been chosen so as to reproduce as nearly as possible the experimental curves of Fig. 47, which are taken from the results of Ewing.* The similarity of the ascending and descending curves, more particularly the outer ones, is very marked.

The principal differences to be noted between the experimental and theoretical curves are, first, that the upper limits of the cycles for medium fields fall more nearly on the outer cycle in the experimental than in the theoretical curves, and second, that for fields but slightly greater than H_o the theoretical curves are rectangular in shape while the experimental curves are not.

32. *Exceptions to the Electron Theory.* — While the electron theory is capable of explaining many of the phenomena of magnetism, yet in its present form and present stage of development it is unable to account for a large number of cases.

Curie's rules, which are the basis of the present theory, hold rigidly for very few substances. Thus, according to these rules the diamagnetic susceptibility is independent of the temperature. However, there are substances whose diamagnetic susceptibility increases with increase of temperature, while in other substances the opposite is the case. Another of Curie's rules states that, for paramagnetic substances, the susceptibility is inversely proportional to the absolute temperature. While this holds for a very large number of substances, there are cases where the rule fails to represent the facts as determined by experiment. Recently H. Kamerling Onnes and A. Perrier† have shown that for several substances the law does not hold for very low temperatures. Some substances at temperatures below those at which Curie's c/T law is obeyed follow more nearly a $\frac{c}{\sqrt{T}}$ law. None of the salts investigated by the above authors show signs of saturation phenomenon. Pyrrhotite, whose magnetic properties conform to the electron theory very closely up to Θ, the temperature of transformation, is very abnormal above this temperature.

From his experimental results upon a limited number of substances, Curie comes to the conclusion that the paramagnetic susceptibility is independent of the state of aggregation of chemical combination of elements. Now oxygen and boron are paramagnetic, oxygen strongly so, yet the oxide of boron is diamagnetic. Likewise Al_2O_3, M_gO, M_oO, ThO, UrO and other oxides are diamagnetic.

The law of approach of the intensity of magnetization to the constant value of saturation holds only for cobalt and not for iron and nickel.

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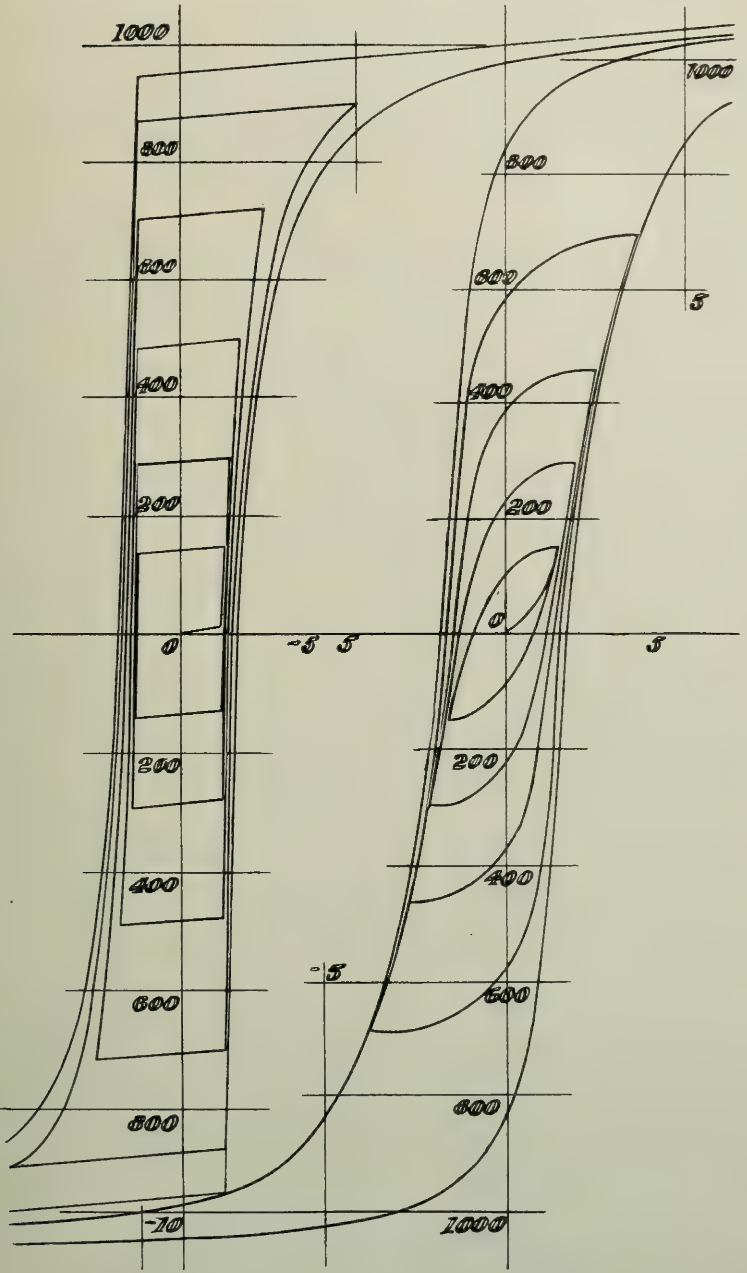
* Ewing, Magnetic Induction, 3rd. Ed., p. 106, Fig. 50.

† H. Kamerling Onnes and A. Perrier, Konink. Akad. Wetensch., Amsterdam Proc. 14, p. 115, 1911.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200
400
-5
600
600
1000

Fig. 46

Fig. 47

The large number of exceptions to the electron theory in its present form requires either that it be abandoned or that the theory be modified to fit more exactly experimental results. The fact that it agrees in such a large number of cases with experiment and that, by its application, the fundamental quantities of nature can be obtained in such close agreement with observation, gives hope that ultimately the present theory will be modified so that it will hold universally.

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